SERBIATRIB '11

Serbian Tribology Society

12th International Conference on Tribology

Faculty of Mechanical Engineering in Kragujevac

Kragujevac, Serbia, 11 – 13 May 2011

INFLUENCE OF FRICTION ON THE FORCE DISTRIBUTION AT CYCLOIDAL SPEED REDUCER

Mirko Blagojević[1], Nenad Marjanović[1], Blaža Stojanović[1], Zorica Đorđević[1], Milorad Kočić[2]
[1]Faculty of Mechanical Engineering in Kragujevac, Serbia, mirkob@kg.ac.rs
[2]Institute Gosa, Smederevska Palanka, Serbia, milorad.kocic@institutgosa.rs

Abstract: *Cycloidal speed reducer belong to the generation of modern planetary gears. Their main features are: high transmission ratio, compact design, long and reliable working life, high efficiency, possibility of accepting big short-term overloads, wide application in various industries,...*
The main element of cycloidal speed reducer is certainly cycloid disc whose profile is equidistant of shortened epitrochoid. Analysis of the forces which act on the cycloid disc when friction is taken into account is presented in this paper. Comparative review of calculated forces in the presence and absence of friction is presented.
The main conclusion arising from the results of of this study are that the phenomenon of friction has an important and unavoidable impact on the load distribution in cycloid speed reducer.

Keywords: *cycloidal speed reducer, cycloid disc, force analysis, friction*

1. INTRODUCTION

Cycloidal speed reducers, have very wild area of application: at conveyors, food machinery, robots, mixers, recycling machines, automotive plants, steel mills, etc. Their main features are: high efficiency, compact design, quite and reliable operation, great gear ratio, minimal vibration, low noise, low backlash, etc. Single-stage cycloidal speed reducer is shown in Figure 1.

M. Lehmann [1] is described the procedure for force distribution at cycloidal speed reducer for theoretical case (when friction don`t exist) in detail. Malhotra [2] is defined some simplified model for force analysis but without friction, too. Distribution of loads at cycloidal speed reducer with modified profile of cycloid disc is presented in papers [3, 4]. Complete geometric and kinematic analysis of cycloid disc tooth profile is described in papers [5, 6, 7]. Gorla, Davoli and other [8] give out the results of theoretical and experimental analysis of cycloidal speed reducer efficiency. Kosse [9] presented the results of experimental study of the hysteresis phenomenon at cycloidal speed reducer and damping properties derived from dickey curves under torsion impact load. Equations for stress, efficiency and moment of inertia of cyclo drives are derived in paper [10]. Stress and strain state at cyclo speed reducer elements is presented in papers [11,12,13]. Meng and other [14] defined a mathematical model of transmission performance of 2K-H pin-cycloid planetary mechanism including the friction.

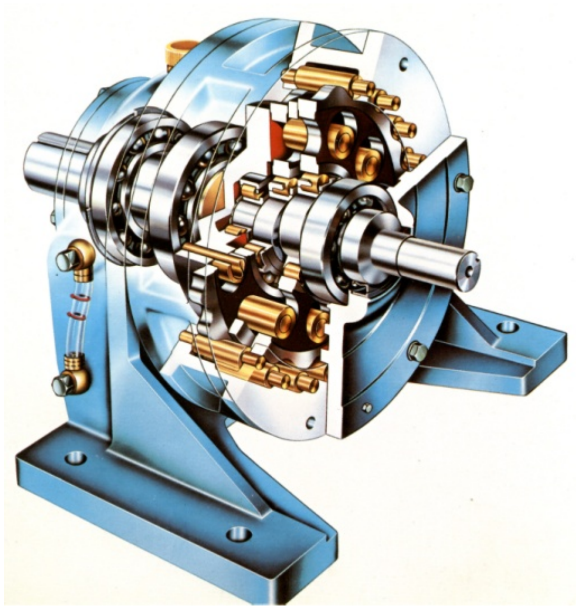

Figure 1. Single-stage cycloidal speed reducer

In this paper is analyzed force distribution at cycloidal speed reducer including the friction.

2. FORCE DISTRIBUTION AT CYCLOIDAL SPEED REDUCER WITHOUT FRICTION

Cycloid disc is the most important element of the cycloidal speed reducer due to its complex geometry and complex stress and strain state. In the first step it is necessary to define forces which act upon it. Cycloid disc with contact elements (housing rollers and output rollers) is shown in Figure 2.

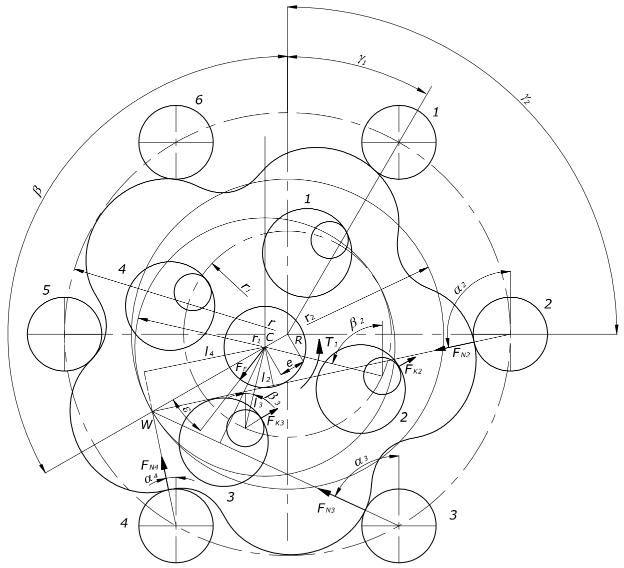

Figure 2. Cycloid disc in contact with housing rollers and output rollers

Forces on cycloid disc are:

F_E – bearing reaction,

F_{Ni} – force between housing roller *i* and cycloid disc,

F_{Kj} – force between output roller *j* and cycloid disc,

T_1 – input torque.

The following equations can be expressed, based on Figure 2:

$$T_1 = F_E \, e \cos(\beta + \varepsilon) \tag{1}$$

$$T_1 = \frac{r_i}{z} \sum_{j=1}^{q} F_{Kj} \sin(\beta_j + \beta) \tag{2}$$

$$-\sum_{i=1}^{p} F_{Ni} \sin\alpha_i + \sum_{j=1}^{q} F_{Kj} \sin\beta - F_E \sin(\beta + \varepsilon) = 0 \tag{3}$$

$$-\sum_{i=1}^{p} F_{Ni} \cos\alpha_i - \sum_{j=1}^{q} F_{Kj} \cos\beta - F_E \cos(\beta + \varepsilon) = 0 \tag{4}$$

$$\sum_{i=1}^{p} F_{Ni} \, l_i - \sum_{j=1}^{q} F_{Kj} \, r_i \sin(\beta_j + \beta) = 0 \tag{5}$$

where:

e – eccentricity,

β – swivel angle of the input shaft,

ε – angle between the force F_E and eccentricity direction,

r_i - radius of output rollers pitch circle,

z - number of teeth of cycloid disk (gearing ratio of the cycloidal speed reducer),

β_j – angular position of the output roller – j ,

α_i – angle which force F_{Ni} makes with vertical,

l_i - lever arm of force F_{Ni} ,

p - number of the housing rollers that carry the load,

q - number of the output rollers that carry the load.

Values α_i and l_i are calculated according to Figure 2, based on the following expressions:

$$\alpha_i = \mathrm{arctg} \frac{\sin\beta + \dfrac{r}{r_2}\sin\gamma_i}{\cos\beta - \dfrac{r}{r_2}\cos\gamma_i} \tag{6}$$

$$l_i = r_1 \sin(\alpha_i - \beta) \tag{7}$$

Angle γ_i (angular position of the housing rollers) is calculated based on the following expression:

$$\gamma_i = \frac{360(2i-1)}{2(z+1)} \tag{8}$$

r - radius of housing rollers pitch circle,

r_1 - base circle radius of the cycloid disc,

r_2 - base circle radius of the housing rollers.

Forces F_{Ni} and F_{Kj} are proportional to their respective distances from the centre of rotation:

$$\frac{F_{Ni}}{l_i} = const. \tag{9}$$

$$\frac{F_{Kj}}{\sin(\beta_j + \beta)} = const. \tag{10}$$

Only for ideal (theoretical) case all cycloid disk teeth are in contact with appropriate rollers and half of them carry load. In reality, cycloidal speed reducer has machining tolerances due to which number of teeth in contact is lower than in ideal case, that is, the load per one tooth is increased.

3. FORCE DISTRIBUTION AT CYCLOIDAL SPEED REDUCER WITH FRICTION

Influence of friction on load distribution at real gear drives is very important. It is the same at cycloidal speed reducer. There are three main areas where friction exist:

- in contact of cycloid disc and central bearing rollers,
- in contact of cycloid disc and output rollers and
- in contact of cycloid disc and stationary ring gear rollers.

Model with friction in contact of cycloid disc and stationary ring gear rollers is analyzed in this paper. Friction in other areas is neglected.

Cycloid disc in contact with output rollers and stationary ring gear rollers (housing rollers) in the presence of friction is presented in Figure 3.

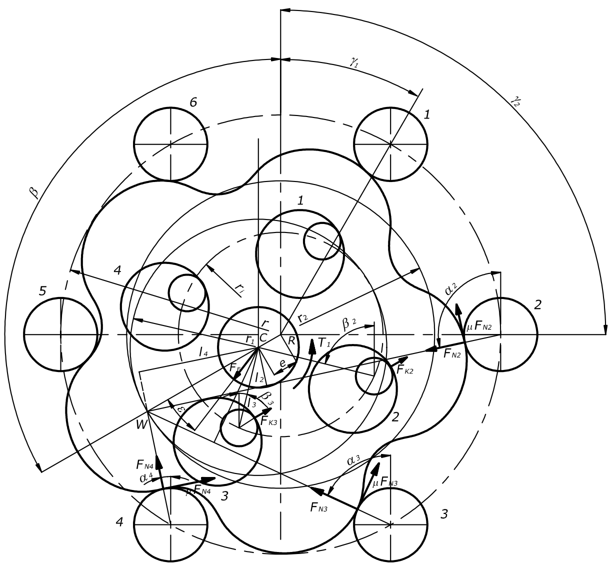

Figure 3. Cycloid disc in contact with housing rollers and output rollers in the presence of friction

The following equations can be expressed, based on Figure 3:

$$-\sum_{i=1}^{p} F_{\mathrm{Ni}} \sin\alpha_{\mathrm{i}} + \sum_{j=1}^{q} F_{\mathrm{Kj}} \sin\beta - F_{\mathrm{E}} \sin(\beta+\varepsilon) - \\ -\mu\sum_{i=1}^{p} F_{\mathrm{Ni}} \cos\alpha_{\mathrm{i}} = 0 \quad (11)$$

$$-\sum_{i=1}^{p} F_{\mathrm{Ni}} \cos\alpha_{\mathrm{i}} - \sum_{j=1}^{q} F_{\mathrm{Kj}} \cos\beta - F_{\mathrm{E}} \cos(\beta+\varepsilon) + \\ +\mu\sum_{i=1}^{p} F_{\mathrm{Ni}} \sin\alpha_{\mathrm{i}} = 0 \quad (12)$$

$$\sum_{i=1}^{p} F_{\mathrm{Ni}}\, l_{\mathrm{i}} - \sum_{j=1}^{q} F_{\mathrm{Kj}}\, r_{\mathrm{i}} \sin(\beta_{\mathrm{j}}+\beta) - \mu\sum_{i=1}^{p} F_{\mathrm{Ni}}\, a_{\mathrm{i}} = 0 \quad (13)$$

where:

μ – coefficient of friction.

Values a_{i} are calculated according to Figure 3, based on the following expression:

$$a_{\mathrm{i}} = r_1 \cos(\alpha_{\mathrm{i}} - \beta) \quad (14)$$

4. ANALYSIS OF THE INFLUENCE OF FRICTION FORCE ON LOAD DISTRIBUTION FOR CONCRETE CYCLOIDAL SPEED REDUCER

Analysis of the influence of the friction force on load distribution was done for concrete one-stage cycloidal speed reducer with next characteristics:

Input power: $P = 1{,}1\,\mathrm{kW}$

Input rpm: $n = 1410\,\mathrm{min}^{-1}$

Gear ratio (number of cycloid disc teeth): $u = 5$ ($z_1 = 5$)

input torque: $T_1 = 3{,}725\,\mathrm{Nm}$

The most critical case of meshing is analyzed in this paper – single meshing.

Based on equations presented in this paper (1-14), calculation of contact force was done. There are four cases: no friction, coefficient of friction $\mu = 0{,}05$; coefficient of friction $\mu = 0{,}1$; coefficient of friction $\mu = 0{,}2$. Results of these calculation are presented in Table 1.

Table 1. Values of contact force

	Contact force F_{N}, N
Without friction	1750
$\mu = 0{,}05$	2109
$\mu = 0{,}1$	2653
$\mu = 0{,}2$	5478

The same results are presented on Figure 4, too.

Based on contact force values for different coefficient of friction (Table 1 and Figure 4), it is clear to see that friction is very important for load distribution at cycloidal speed reducer. With increasing of coefficient of friction, contact force increase, too.

For the real influence of friction on the load distribution it is necessary to take into account and friction between cycloid disc and central bearing rollers and cycloid disc and output rollers.

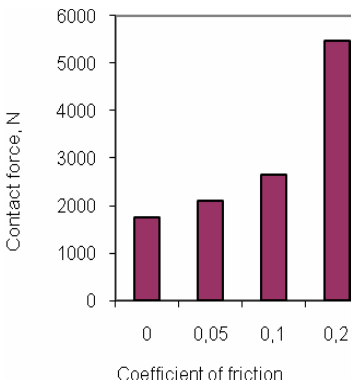

Figure 4. Influence of coefficient of friction on contact force

Taking into account the real lubrication conditions at cycloidal speed reducer, it can be considered that real value of coefficient of friction is $\mu = 0,05$. Further, it means that increasing of contact force for single meshing when friction exist is about 20%.

5. CONCLUSION

Regardless of good lubrication conditions, influence of friction on load distribution is very important. Analytical model for calculating of forces when friction exist is developed in this paper. Friction between cycloid disc and ring gear rollers was analyzed, while friction in other areas was neglected. Some expressions for calculating of contact forces were defined, too.

Calculating of contact force for the most critical case – single meshing for concrete one stage cycloidal speed reducer was done, too. Coefficient of friction is varied in interval 0 to 0,2. The results indicate the occurrence of increasing of contact force with increasing of coefficient of friction.

In the next investigations, it will be interesting to take into account the friction in all areas. It would be very interesting to do the stress and strain analysis with the newly obtained contact forces values.

REFERENCES

[1] M. Lehmann: Calculation and measurement of forces acting on cycloidal speed reducer (in German), PhD Thesis, Technical University Munich, Germany, 1976.

[2] S.K. Malhotra, M.A. Parameswaran: Analysis of a cycloidal speed reducer, Mechanism and Machine Theory, Vol.18, No 6, pp. 491-499, 1983.

[3] L. Lixing, L. Xin, H. Weidong, Q. Yuanmei: Profile modification and accurate force analysis on cycloid drive, in: 4th World Congress on Gearing and Power Transmission, 1999, Paris, France, Vol. 3, pp. 1141-1146.

[4] M. Chmurawa, A. Lokiec: Distribution of loads in cycloidal planetary gear (CYCLO) including modification of equidistant, in: 16th Europian ADAMS user conference, Berchtesgaden, Germany, 2001.

[5] B.K. Chen, T.T. Fang, C.Y. Li, S.Y.Wang: Gear geometry of cycloid drives, Science in China Series E: Technological Sciences, Vol. 51, No. 5, pp. 598-610, 2008.

[6] Y.W. Hwang, C.F. Hsieh: Geometry design and analysis for trochoidal-type speed reducers: with conjugate envelopes, Transactions of the CSME/de la SCGM, Vol. 30, No. 2, pp. 261-278, 2006.

[7] C. Chen, X. Zhang, J. Angeles: Kinematic and geometric analysis of a pure-rolling epicyclic train, Journal of Mechanical Design (ASME), Vol. 129, pp. 852-857, 2007.

[8] C. Gorla, P. Davoli, F. Rosa, C. Longoni, F. Chiozzi, A. Samarani: Theoretical and experimental analysis of a cycloidal speed reducer, Journal of Mechanical Design (ASME), Vol. 130, pp. 112604-1-112604-8, 2008.

[9] V. Kosse: Using hysteresis loop and torsional shock loading to asses damping and efficiency of cyclo drives, 14th International Congress on Sound and Vibration, 9-12. July, 2007, Cairns, Australia.

[10] J. Sensiger: Unified Approach to Cycloid Drive Profile, Stress, and Efficiency Optimization, Journal of Mechanical Design (ASME), Vol. 132, pp. 024502-1-024502-5, 2010.

[11] M. Blagojevic, N. Marjanovic: The force distribution on two stage cyclo speed reducer with new concept, in: 9th International Conference on Accomplishements in Electrical and Mechanical Engineering and Information Technology, Banja Luka, pp. 51-56, 2007.

[12] M. Blagojevic: Stress and strain state of cyclo speed reducer`s elements under dynamic loads, PhD Thesis, Faculty of Mechanical Engineering Kragujevac, Serbia, 2008.

[13] M. Blagojevic, V. Nikolic, N. Marjanovic, Lj. Veljovic: Analysis of cycloid drive dynamic behavior, Scientific Technical Review, Vol.LIX, No. 1, pp. 52-56, 2009.

[14] Y. Meng, C. Wu, L. Ling: Methematical modelling of the transmission performance of 2K-H pin cycloid planetary mechanism, Mechanism and Machine Theory, Vol. 42, pp. 776-790, 2007.