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# SPECIFIC SLIDING OF THE TROCHOIDAL GEARING AT THE GEROTOR PUMP

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***Abstract:*** *In the paper is given the method to determination of the gear geometrical dimensions at the gerotor pump on the base of the specific sliding equality in the point with the greatest sliding velocity. The gerotor pumps are the rotated pumps with internal trochoidal gearing. The pump working process is followed with mechanical losses as a consequence of the wearing between the surfaces of the meshing tooth profiles. To reduce these unwanted effects and taking out the gear active tooth surfaces wearing is necessary to realize minimum of the film oil between contact surfaces. Minimum oil film thickness is in function from the numerous different factors and in this paper are analyzed only geometrical and kinematical characteristics of the meshing tooth profile surfaces. At first are defined the geometrical and functional working limits for the gears. It is developed the mathematical model in form of the parameter equations which is following the change of the relative velocities in contact, sliding velocity, rolling velocity and specific sliding of the gear tooth profiles in the contact point. They are defined the conditions for the equality of the specific sliding in the critical contact points. To the observed geometrical-kinematical model of the gerotor pump is developed the computer program and on the base of the obtained results are chosen the values of the parameters to the realisation of the concrete teeth pairs of the gerotor pump with the better tribological characteristics.*

***Keywords:*** *gerotor pump, trochoidal gearing, sliding velocity, specific sliding.*

## 1. INTRODUCTION

The basic components of the most contemporary machines are gears and oft their quality defined the life time and reliability of machine. The often causes of the functional incorrectness at the gear pairs are different types of the toot flanks wear. From this point of view is very important that to the exploitation and technological demands it is necessary to be chosen corresponding gearing profile at the applied gears, which would be most suitable to its construction form taking into consideration contact stresses and tooth flanks wear. To reduce these unwanted effects and taking out the gear active tooth surfaces wearing is necessary to realize minimum of the film oil between contact surfaces. Minimum oil film thickness is in function from the numerous different factors and in this paper are analyzed only geometrical and kinematical characteristics of the meshing tooth profile surfaces.

In the present time are using most often the gears with involute gearing that besides the great technical and economical advances has also many deficiencies. The main deficiency is the relative small carrying capacity of the tooth flanks and the great contact stresses and it comes to the crease of the efficiency coefficient. The good base to the solutions of the most problems of the gear pair is giving trochoidal gearing. Using this gearing in contemporary constructions can be supplied the basic functional characteristics: needed efficiency coefficient and working life with minimum weight and dimensions. Also at the trochoidal gearing is realised in the same time contact of the all tooth with them can be attained better caring capacity.

In the last time engineers and researches indicated the great interesting to the development and application of the trochoidal form. The most known examples of the trochoidal profiles application are: rotated pumps, rotated motors, rotated compressors, expanders and cycloreducers.

Ansdale and Lokley have done the equations that defined geometry of trochoidal profile, used in construction of Wankel motor [1]. Litvin is applied the gearing theory for generating cycloidal gearing [2]. Maiti has given the detailed analyze of geometrical, kinematical and functionality characteristics of rotated machines with the gerotor mechanism [3]. Beard and other authors investigated the kinematical and geometrical relations between the gear and the cutting tool during the profile generation process is the same relation [4]. Mancò and other authors are developed the computer method for generating gerotor lubricating pump [5].

The existing construction solution on the lubrication pump gears is given in Fig.1. The main part of the gerotor pump is a gear pair which is assembled to the main shaft by a gear key. The pump body has fluid outlets which are connected to pump suction line and delivery line. The cover is on the front side. Both gears rotate counterclockwise. During the gear rotation the inlet chambers are in the area where the teeth are coming out from the tooth space and the outlet chambers where they are coming in contact in the teeth space.

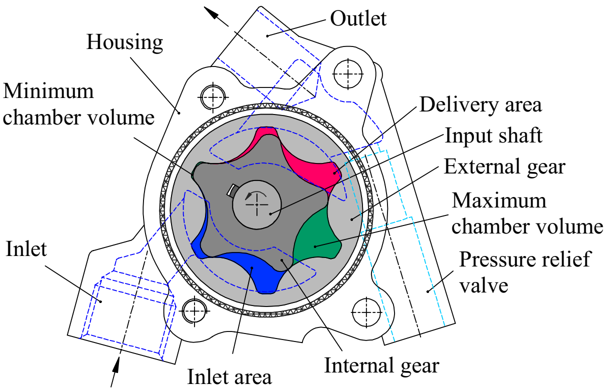



**Figure 1.** The scheme of the pump model

The functional characteristics of the pump are significantly influenced by the gear pair parameters. This paper will analyze the influence of geometrical and kinematical parameters of the gear pair profile on the pump functional characteristics and their influence on the specific sliding.

## 2. GEOMETRYCAL PARAMETERS AND LIMITS

At the pump projection is given usually constructive parameters: eccentricity $e$, teeth number of the external gear $z$ and radius of the root circle of the external gear $r_{fa}$ of the mashing envelope (Fig.2). it is necessary to be defined the limit value of the radius $r_c$ which at the given value trochoid coefficient $\lambda$, does not come to the undercutting and interference of the toot profile [..]. This analyze will be done on the base of the geometrical relations given in the Figure 2.

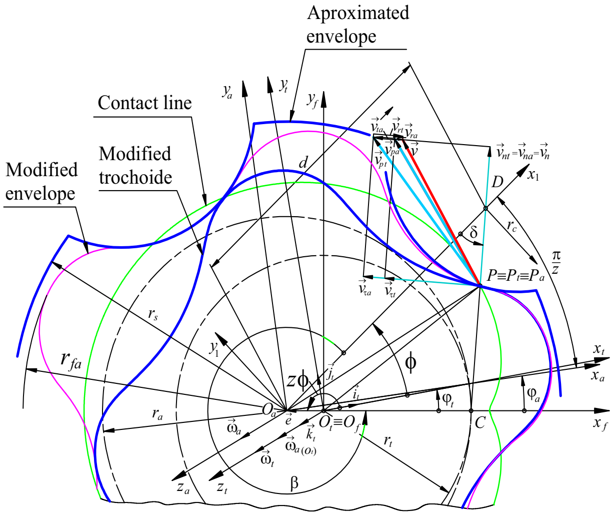



**Figure 2.** Geometrical and kinematical parameters of the trochoidal gear pair

Starting of equations that defined coordinates of the contact point $P$ in coordinate system of trochoide:

$$x_t = e(\cos z\phi + \lambda z \cos\phi) - r_c \cos(\phi + \delta)$$
$$y_t = e(\sin z\phi + \lambda z \sin\phi) - r_c \sin(\phi + \delta) \quad , \quad (1)$$

by them $\delta$ is leaning angle defined to the following form:

$$\delta = \arctan \frac{\sin(z-1)\phi}{\lambda + \cos(z-1)\phi} . \quad (2)$$

To be realized meshing gear regulation of the trochoidal tooth pairs is necessary to be filled numerous geometrical and kinematical conditions. In this part of paper are analyzed the conditions that come to the appearance undercutting and interference of tooth profiles.

Criteria for the choice of equidistant radius, to eliminate the phenomenon of tooth profile undercutting is

$$c < z\sqrt{\left(\frac{3}{z+1}\right)^3 (\lambda^2 - 1)(z-1)} , \quad (3)$$

where $c$ is equidistante coeficient $c = r_c/e$ , and condition for elimination of interference of neighboring tooth profiles of external gear is given

$$c \le z\lambda \sin\frac{\pi}{z} . \quad (4)$$

To be realized the regular meshing of the convex profile part of the modified trochoide in the root area of the meshing envelope that is approximated with circular arc, it is necessary to be filled condition for:

$$r_s > r_{fa}\,, \tag{5}$$

by them is

$$r_{fa} = e(\lambda z + 2) - r_c\,. \tag{6}$$

Taking in coefficient $S_{fa} = r_{fa}/e$ from equation (6) can be defined radius of the equidistance in form

$$c = \lambda z + 2 - S_{fa}\,. \tag{7}$$

To be got correct manufacturing and montage it is needed that $r_s$ is less than $d = e\lambda z$ and can be expressed with following condition:

$$c > 2\,. \tag{8}$$

The value of the radius equidistance for the given constructive parameters is chosen from interval

$$c_{\min} < c < c_{\max}\,, \tag{9}$$

by them is taken for $c_{\min}$ more value from the values obtained to the formulae (7) and (8), and for $c_{\max}$ is taken less value from the values obtained to the formulae (3) and (4),

Starting from analytical relations realized for given geometrical limits can be defined domain of the practical application analyzed geometrical parameters for the internal gear pairs with modified trochoidal gearing.

## 3. KINEMATICAL PARAMETERS

To the kinematical analyzes of the meshing profiles is considerate the complex moving of the point $P_t$ on the internal gear profile and the point $P_a$ on the external gear profile. There is supposition that is contact in some moment at the points $P_t$ and $P_a$ in the point $P$ on the contact line, as is shown in the Figure 2. For the correct meshing and their continuality is necessary to be realized the condition about equality of the absolute velocities of the meshing profiles in the contact point as also their components in the common normal direction.

From gearing theory it is known that only centroide can realize rolling without sliding. According to, profile sliding is inevitable because they are formed with the curves which are different of centroide. The sliding velocity of the meshing profiles $\vec{v}_{ta}$ in the observed contact point is the velocity of the contact point at the relative profile moving and it is defined with the difference at the transfer velocity vectors, regard to

$$\vec{v}_{ta} = \vec{v}_{pt} - \vec{v}_{pa}\,. \tag{10}$$

The angular frequency of the internal and external gear are defined with the vectors $\vec{\omega}_t$ and $\vec{\omega}_a$. To determination of the relative moving velocity $\vec{v}_{ta}$ vector $\vec{\omega}_a$ is coming through the parallel moving in the point $O_t$ [2]. As the result of reduction is obtained vector $\vec{\omega}_{a(O_t)}$, which is acting in the point $O_t$, and the vector moment is defined the following vector equation:

$$\vec{m}_a^{(t)} = \vec{e} \times \vec{\omega}_a\,, \tag{11}$$

by them is $\vec{e}$ radius vector of point trough which is acting vector $\vec{\omega}_a$ in the coordinate system $O_t x_t y_t z_t$ (Fig. 2), which can be written in the form:

$$\vec{e} = -e\cos\varphi_t \vec{i}_t + e\sin\varphi_t \vec{j}_t\,. \tag{12}$$

Transfer velocity vectors of the point $P_t$ can be done in form of relation:

$$\vec{v}_{pt} = \vec{\omega}_t \times \vec{r}_t = \begin{vmatrix} \vec{i}_t & \vec{j}_t & \vec{k}_t \\ 0 & 0 & \omega_t \\ x_t & y_t & 0 \end{vmatrix} = -y_t\omega_t\vec{i}_t + x_t\omega_t\vec{j}_t\,. \tag{13}$$

by them are $\vec{i}_t, \vec{j}_t, \vec{k}_t$ unit vectors of the coordinate system of the trochoide $O_t x_t y_t z_t$.

Transfer velocity vectors of the point $P_a$ can be shown in the form:

$$\begin{aligned} \vec{v}_{pa}^{(t)} &= \vec{\omega}_{a(O_t)} \times \vec{r}_t + \vec{e} \times \vec{\omega}_a = \\ &\begin{vmatrix} \vec{i}_t & \vec{j}_t & \vec{k}_t \\ 0 & 0 & \omega_a \\ x_t & y_t & 0 \end{vmatrix} + \begin{vmatrix} \vec{i}_t & \vec{j}_t & \vec{k}_t \\ -e\cos\varphi_t & e\sin\varphi_t & 0 \\ 0 & 0 & \omega_a \end{vmatrix} = \\ &= (-y_t + e\sin\varphi_t)\omega_a\vec{i}_t + (x_t + e\cos\varphi_t)\omega_a\vec{j}_t \end{aligned} \tag{14}$$

According to, the vector of the profile sliding of the internal in relation to external gear, to the equation (10), in coordinate system of trochoide can be given in the form of vector equation:

$$\begin{aligned} \vec{v}_{ta}^{(t)} &= \left\{-e(\sin z\phi + \lambda\sin\phi) + \frac{r_c}{z}\sin(\phi+\delta)\right\}\omega_t\vec{i}_t \\ &+ \left\{e(\cos z\phi + \lambda\cos\phi) - \frac{r_c}{z}\cos(\phi+\delta)\right\}\omega_t\vec{j}_t \end{aligned} \tag{15}$$

Intensity of the profile sliding velocity in the contact point is:

$$v_r = |\vec{v}_{ta}| = \left\{\left(1 + \lambda^2 + 2\lambda\cos\beta\right)^{\frac{1}{2}} - \frac{c}{z}\right\} e\omega_t\,, \tag{16}$$

where

$$\beta = (z-1)\phi\,. \quad (17)$$

Except the sliding velocity for the analyzing of the phenomenon of wear significant is the summary rolling velocity [5]. Intensity of summary rolling velocity can be written in the form:

$$v_{\Sigma} = \left\{ \left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}} - \frac{c}{z}\left(1+2\delta'\right) \right\} e\omega_t\,, \quad (18)$$

where

$$\delta' = \frac{(z-1)(1+\lambda\cos\beta)}{1+\lambda^2+2\lambda\cos\beta}\,. \quad (19)$$

The formulae to determination the specific sliding of the meshing profiles in the contact point can be defined on the base of the obtained equations.

## 4. SPECIFIC PROFILE SLIDING

The sliding existence in the sliding in the meshing profiles process comes to their wear by that the sliding velocities define the friction forces direction and intensity which take effect on the meshing profiles of the gears. By them the friction force is in direction opposite to the relative motion velocity in the contact point. So the direction of the sliding velocity $\vec{v}_{ta}$ is in agreement with the direction of the friction force which take effect on the profile of the external gear profile, but the direction of $\vec{v}_{at}$ is the same the direction of the friction force on the trochoidal profile [12].

It is necessary to know, at the analysis of the meshing profiles sliding, except the sliding velocity in the contact point also its distribution of their change in relation to corresponding relative velocity of the contact point. Specific sliding is relation between the sliding velocity and relative velocity of the contact point of the meshing profiles [7]. After the substitution of the corresponding formulae for the velocities is obtained finally formulae for the specific sliding on the tooth profile of the internal gear:

$$\xi_t = \frac{z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}} - c}{z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}} - c\left(1+\delta'\right)}\,, \quad (20)$$

and analogical for external gear:

$$\xi_a = \frac{z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}} - c}{c\delta'}\,. \quad (21)$$

Specific sliding is positive on the profile point, where the directions of sliding and relative velocities are in agreement, and where are not in agreement – negative.

### 4.1 Conditions for the uniform teeth wear

Influence of geometrical and kinematical parameters on the sliding value and intensity of the teeth profile wear is analyzing through the specific profile sliding [7], [8]. The aim of constructer is that with the corresponding choice of geometrical parameters can be realized the uniform wear of the meshing gear teeth in the meshing process. From this aspect is necessary that it is realized equality of the specific sliding in the points with the greatest of the pitch point of the relative velocities. Starting from the formulae (20) and (21), taking in also (19), comes that for the given values of the teeth number $z$ and coefficient of trochoide $\lambda$ the uniform wear of the tooth profile can be realized when is filled condition about equality of the relative velocities, in regard to, when is chosen value of the coefficient of equidistant radius equal:

$$c = \frac{z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{3}{2}}}{\lambda^2-1+2z\left(1+\lambda\cos\beta\right)}\,. \quad (22)$$

This condition can be written for the points with the greatest sliding velocity, and they are the points on the top of the trochoide profile ($\beta$=0), in the form:

$$c = \frac{z(1+\lambda)^2}{2z+\lambda-1}\,. \quad (23)$$

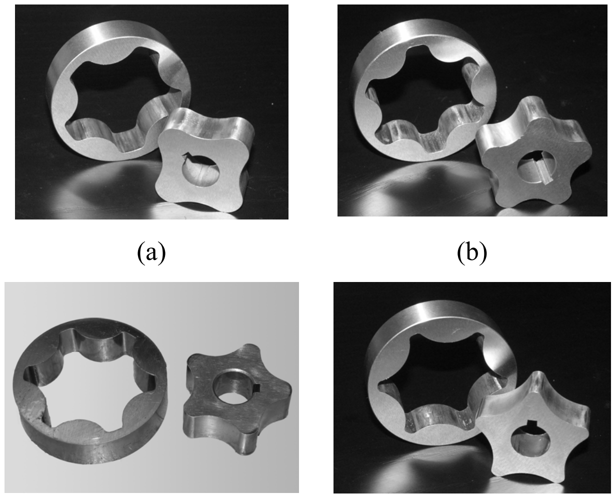
## 5. NUMERICAL EXAMPLES

On the base of the realized analyze as result of the developed methodology for the identification of the optimal geometrical parameters of the trochoidal pump [9], are proposed the solutions with the parameter which are given in the Table 1.

**Tabela 1.** Parameters of various pump models

| Pump parameters | Number of pump chambers | | | |
|---|---|---|---|---|
| | $z$=6 | | | $z$=5 |
| Trochoid coefficient, $\lambda$ | 1.375 | 1.575 | 1.675 | 1.85 |
| Equidistant radius coefficient, $c$ | 2.75 | 3.95 | 4.55 | 3.75 |

In the Fig. 3 are given alternative solutions of the gear pairs.



(a) (b)

(c) (d)

**Figure 3.** Alternative solutions of the gear pairs theoretical profiles: a) GP-850, b) GP-375, c) GP-575 (commercial) and d) GP-675

This part will present some of the results of the kinematical analysis of trochoidal gearing on the concrete examples of gear pairs of the investigated pump models.

In the Figure 4 are shown diagrams of the sliding velocity $v_r$ and in the Figure 5 summary rolling profile velocity $v_\Sigma$ in relation of the angle $\beta$. Because that the trochoide is cyclical curve with the symmetrical branches, by them is one half generated in angular interval $0 \leq \beta \leq \pi$, that is mean that the obtained diagrams give the clear picture about distribution of the velocity changing along the profile, by them for the top of the trochoidal profile is corresponding angle $\beta$=0, and in the root $\beta=\pi$.

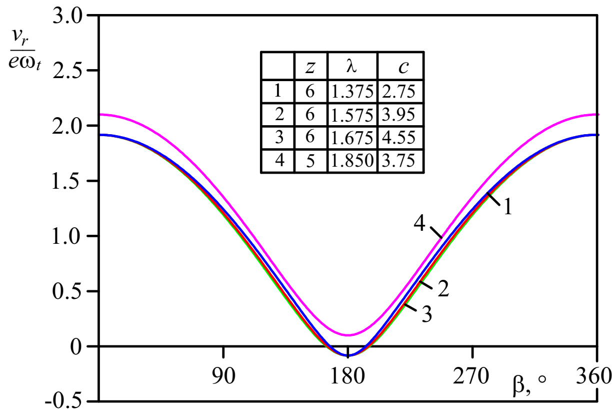



**Figure 4.** Sliding velocity $v_r$ of the contact point of trochoidal profiles for different teeth number $z$, values of the trochoid coefficient $\lambda$ and coefficient of equidistant radius $c$

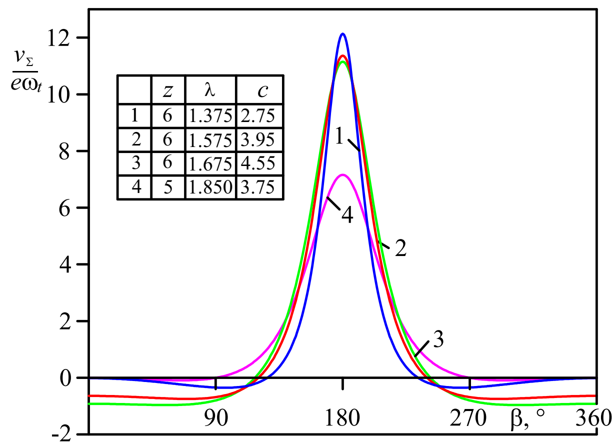
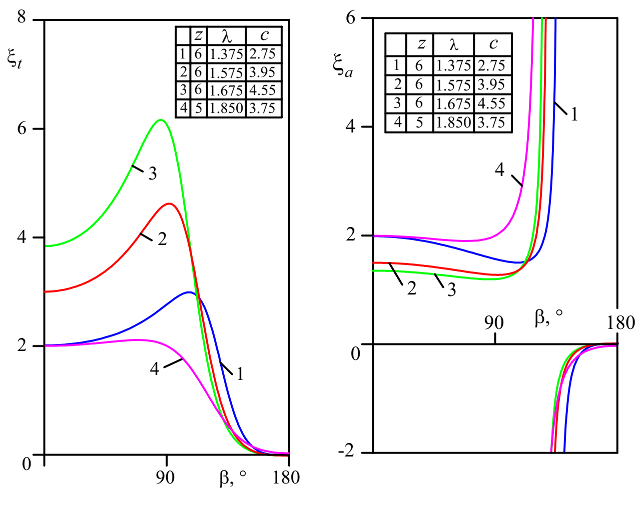



**Figure 5.** Summary rolling velocity $v_\Sigma$ of the contact point of trochoidal profiles for different teeth number $z$, values of the trochoid coefficient $\lambda$ and coefficient of equidistant radius $c$

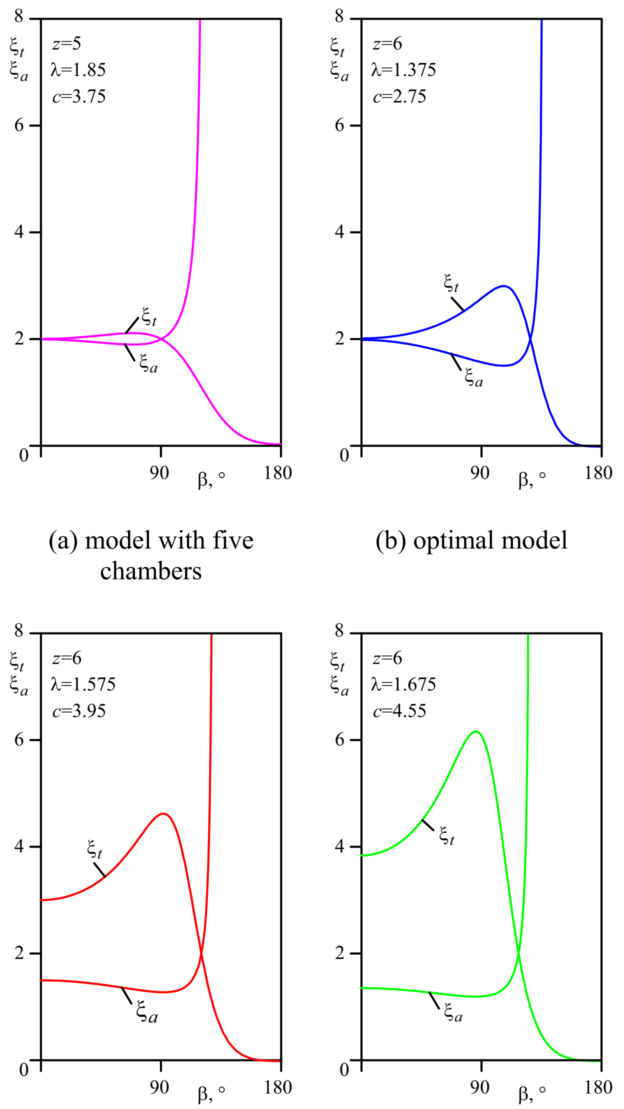
On the base given graphical interpretation obtained results can be defined the following conclusions:

- At the model with $z$=6 the sliding velocity $v_r$ monoton decrease from maximal (on the top), is equal zero in the moment when the profile are in contact in pitch point abd by them comes to the sign change;
- At the model with $z$=5 the sliding velocity $v_r$ monoton decrease from maximal (on the top) to the minimal value (in the root of the profile);
- Summary rolling velocity $v_\Sigma$ has aproximate constant value, near zero (on the convex part of the profile), and after that abruptly increases to the maximal value (in the root of the concave part of the profile).




(a) (b)

**Figure 6.** Comparative diagrams of specific sliding of trochoid profiles




(a) model with five chambers

(b) optimal model

(c) commercial model

(d) comparative model

**Figure 7.** Diagrams of specific sliding trochoid profiles

The sign change of the sliding velocity and the summary rolling velocity has influence on the direction change of the moment of the force sliding friction and the force of the rolling friction at the determination of the mechanical power losses [9].

Figure 6 and 7 illustrates comparative diagrams of specific sliding profile, on the basis of which the following conclusions can be drawn. Specific sliding on the convex part of trochoidal profile significantly decreases at the chosen solutions, while with the circular profiles of external gears it increases at the top part, at the chosen solutions. It is verified that the conditions for even wear of profiles at the point with the highest sliding velocity are met at the chosen solutions, from the aspect of equality of specific sliding.

## 6. CONCLUSION

In this paper are defined the limits at the profile generating to the aim to establish domains of the practical application of the geometrical parameters of the gear pair. From this aspect are considered conditions that come to appear of undercutting and interference of the profile as at their manufacturing so that in the process of meshing.

In the paper is given a analysis of the kinematical parameters of the trochoidal gearing. There are defined the formulae to calculation of the sliding and rolling velocities in the contact points of the meshing profiles. The paper also gives a analysis of specific sliding at the contact points of the meshing profiles, as well as the relations for its determination. In the choice of the geometrical parameters of the conditions for appearing friction and wear of the contact gear surface specific sliding is one of the more important limiting factors. Based on mutual relations of the sliding and rolling velocities values, conclusions can be made about the changing of the friction conditions during the meshing of profiles.

Except that, in this paper is shown that with the corresponding choice of the geometrical parameters can be realized the equality of the specific sliding in the points with the greatest sliding velocity, in regard to, from the kinematical aspect, ensured the uniform teeth wear of the meshing gears in the process of the meshing profiles.

The developed kinematical model is most helpful in designing, contact and stress analyzing, manufacturing and optimizing internal trochoidal gear pairs.

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