*Contacts in gearing*

# TRIBOLOGICAL ASPECT OF THE KINEMATICAL ANALYSIS AT TROCHOIDAL GEARING IN CONTACT

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## ABSTRACT

The generating and kinematical analysis of the gear pair profiles in contact with internal trochoidal gearing from tribological aspect is considered in this paper. By the working elements of the rotational machines is used trochoidal gearing, as for an example, at gerotor pumps and hydro motors. In this paper are analysed the geometrical and kinematical relations of the trochoidal gearing the specific sliding in the meshing profiles process and the numerical examples of the influence of the different construction parameters on the kinematical characteristics of the trochoidal gearing in contact. By them are taken the different coordinate systems and with the application of coordinates transformation is possibly to realise the most simple equations. The moving of the contact point of two profiles is considered in the kinematical analysis as complex because during the contact period the profiles are rolling and sliding in the same time in relation ones to other. Using geometrical and kinematical models of the trochoidal gearing, we provide detailed analysis of specific sliding at the contact points of the profiles, as well as the relations for its determination. To the analysis of the wear phenomenon except of the sliding velocity is important also the size of the summary rolling velocity. Based on the graphical interpretation of kinematical parameters profiles in contact of trochoidal gear pair can be concluded about the influence of geometrical parameters on the size of sliding and also on the wear intensity of the teeth profile.

*Keywords*: trochoidal gearing, sliding velocity, specific sliding, wear intensity.

## AIMS AND BACKGROUND

Theoretical model of the gearing with modified trochoidal profile is applied to the gerotor pumps. Gerotor pumps belong to the group of the planetary rotating ma-

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chines which kinematic is based on the principle of the planetary mechanism with the internal gearing. By them is made classification of rotational machines in two main groups: (1) with fixed axis of the working elements, and (2) with planetary moving of the internal elements. Kinematical analysis can be done in conditions which are valid for one type of machine, and later all kinematical relations can be realised by the concept of equivalent system. The principle of the generating teeth profiles in contact is the same to the working principle of the given second group. By this gearing teeth number of the external gear is for one more of the teeth number of the internal gear. In this case of gearing, moving circle is rolling without sliding along the other stationary circle and by them chosen point describes the trochoidal tooth profile[1,2]. To the stationary circle is, conditionally taken, kinematical circle of the gear. By the relative moving the meshing profile is presented as the envelope of the based profile successive position. In the general case the meshing envelope has peaks, which are unwanted because is coming to the intensive wear and that would be get out of their phenomena is introduced the modification of the based trochoid. Through the increase in constant value of $r_c$, presented in Fig. 1, which is putting along the normal on the given curve, are modified the trochoidal curves. The obtained curve is equidistance and the constant increase of $r_c$ can be defined as the radius of equidistance.

On the basis of geometrical and kinematical models which are developed in Refs 3–10, in this paper will be defined the formulae for the calculation of the based kinematical parameters. To this aim are introduced the following coordinate systems: generating coordinate system, connected for generating point, coordinate system of trochoid, coordinate system of envelope and stationary coordinate system. To the kinematical analysis of the meshing profiles is considered the moving of the contact point of the meshing profiles.

## GEOMETRICAL AND KINEMATICAL RELATIONS OF THE TROCHOIDAL GEARING

In Fig. 1 are given the based geometrical relations of the generating of the unmodified and modified epitrochoid. The point D fixed in the circle plane with the radius $r_a$ describes epitrochoid when the circle with its internal side is rolling along the external side of the stationary circle with its radius $r_t$. The equations of the equidistance epitrochoid are given on the coordinate system of the trochoid $O_t x_t y_t$ to Fig. 1.

During the relative moving of the kinematical circles, when the point D is generating epitrochoid in the same time, the point P is generating equidistance. By them the angle signified with δ presented the angle between normal *n–n* and radius vector of the point D and can be defined as the leaning angle. Coordinates

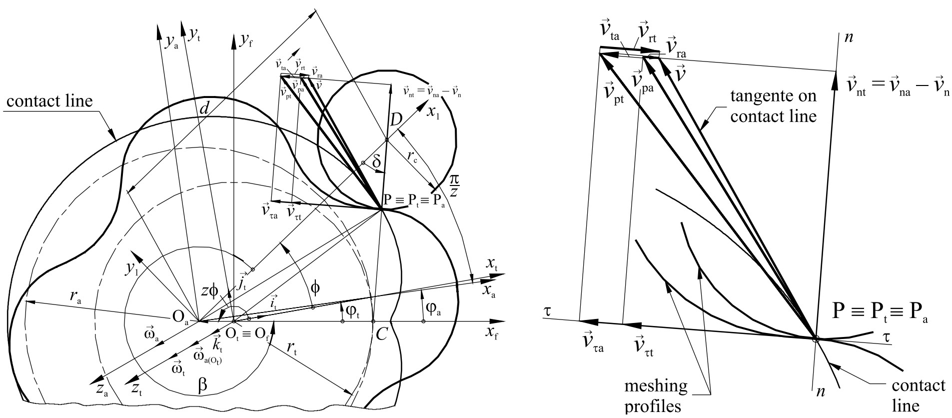


**Fig. 1**. Geometrical and kinematical parameters of the trochoidal gear pair

of the point P in the coordinate system of the trochoid can be written in the form:

$$x_t = e\left[\left(\cos z\phi + \lambda z \cos\phi\right) - c\cos\left(\phi + \delta\right)\right],$$
$$y_t = e\left[\left(\sin z\phi + \lambda z \sin\phi\right) - c\sin\left(\phi + \delta\right)\right] \quad (1)$$

where λ is coefficient of the trochoid which defines the relation between the values of trochoid radius and the radius of the moving circle $\lambda = d/ez$, $c$ – coefficient of the equidistance $c = r_c/e$.

Using the geometrical relations from Fig. 1, the formula for determination of angle δ can be obtained as follows:

$$\delta = \arctan \frac{\sin(z-1)\phi}{\lambda + \cos(z-1)\phi}. \quad (2)$$

For the kinematic analysis of the meshing profiles is considered the moving of the point $P_t$ on the profile of the internal gear and the point $P_a$ on the profile of the external gear (Fig. 1).

During the meshing the trochoidal gearing profiles are rolling in the same time and sliding one to each other. The sliding of the profiles in the contact point is a consequence of the difference of the relative velocities intensity of the points on the profiles of the internal, respectively the external gear. It is known from the gearing theory that only pitch circles can realise rolling without sliding. Based on this is coming out that the profile sliding is inevitable because they are made with the curves which are differentiating than pitch circles. In this case the sliding velocity of the meshing profiles in the observed contact point is velocity of the contact point by the relative profiles moving.

The distribution of the velocity in the contact point of the two meshing profiles is given in Fig. 1 (Refs 3 and 4), where $\vec{v}$ is vector of the absolute velocity of the meshing profiles in the contact point P; $\vec{v}_{pt}$, $\vec{v}_{pa}$ – vectors of the transfer velocities of the contact point $P_t$, $P_a$; $\vec{v}_{nt}$, $\vec{v}_{na}$ – projections of the transfer velocity on the common normal and $\vec{v}_{\tau t}$, $\vec{v}_{\tau a}$ on the tangent in the contact point $P_t$, $P_a$; $\vec{v}_{ra}$ – vectors of the relative velocities of the contact point $P_t$, $P_a$; $\omega_r = \omega_t - \omega_a$ – angular velocity of the epitrochoid in relation to the envelope; $\vec{v}_{ta}$ – vector of the sliding velocity of the internal gear profiles in relation to external gear profiles; $\vec{v}_{at}$ – vector of the sliding velocity of the external gear profiles in relation to internal gear profiles.

The intensity of the relative velocity vector in the contact point $P_t$ of the profile is:

$$v_{rt} = \left\{ z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}} - c\left(1+\delta'\right) \right\} e\omega_r, \tag{3}$$

and intensity of the relative velocity vector in the point $P_a$ is equal to:

$$v_{ra} = ec\delta'\omega_r, \tag{4}$$

where $\delta'$ is:

$$\delta' = \frac{d\delta}{d\phi} = \frac{(z-1)(1+\lambda\cos\beta)}{1+\lambda^2+2\lambda\cos\beta}. \tag{5}$$

The intensity of the profile sliding velocity in the contact point is as follows:

$$v_r = \left|\vec{v}_{ta}\right| = \left\{ z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}} - c \right\} e\omega_r. \tag{6}$$

Sliding velocity is equal to zero only when the profiles are in contact in the pitch point and by them is coming to sign changing. At the trochoidal gearing contact line of the profiles is not going always through the pitch point. In Fig. 2 are shown two different forms of the contact line: (*a*) contact line is smooth curve

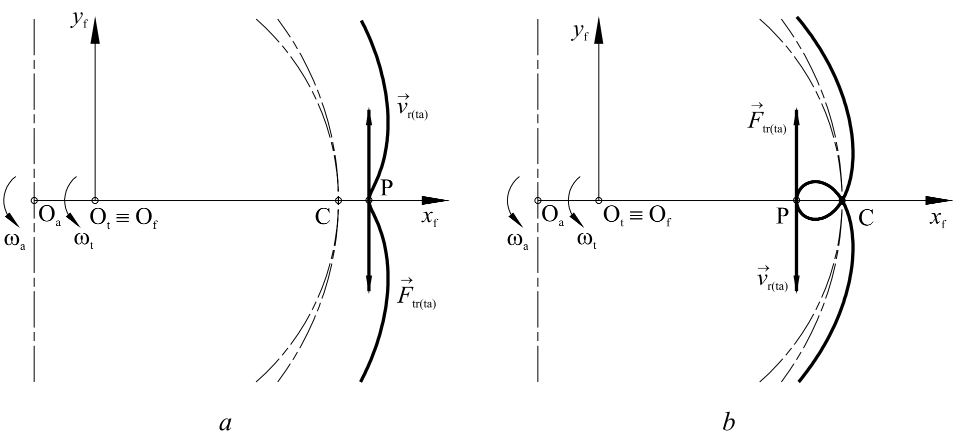


**Fig. 2**. Direction of the sliding velocity and friction force in the contact point P on the referent line at the different form of the contact line of the trochoidal gear pair

and does not go through the pitch point, (*b*) contact line is going through the pitch point and has loop.

If the distance of the contact point from the pitch point is presented in the form:

$$\overline{\mathrm{CP}} = e\left[ z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}} - c \right], \tag{7}$$

based on formulae (6) it can be concluded that the sliding velocity has linear increase with the enlargement at distance $\overline{\mathrm{CP}}$. Because of that, from point of view from the practical using, is desirable that the contact points are in the surroundings of the corresponding kinematical circles.

Starting of equation (7) it can be established by which combinations of the geometrical parameters is coming to the phenomenon of the loop and changing of the sliding velocity sign. The distance of the contact point from the pitch point is equal to zero when:

$$c = z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}}. \tag{8}$$

In this moment to the contact of the trochoidal profile is corresponding to the angle $\beta=\pi$ (root of the profile), so that the condition for the loop appearance of the contact line of the profile is obtained:

$$c > z(\lambda-1). \tag{9}$$

The obtained formula is significant for determination of the torque change direction of the sliding friction force at the determination of mechanical power losses (Fig. 2).

Except the sliding velocity for analysing of the phenomenon of wear is significant the summary rolling velocity. Intensity of summary rolling velocity can be written in the form:

$$v_\Sigma = \left\{ z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}} - c\left(1+2\delta'\right) \right\} e\omega_{\mathrm{r}}. \tag{10}$$

The formulae for determination of the specific sliding of the meshing profiles in the contact point can be defined on the basis of the obtained equations (Fig. 1).

## SPECIFIC PROFILE SLIDING

The sliding existence in the sliding in the meshing profiles process comes to their wear by that the sliding velocities define the friction forces direction and intensity which affect the meshing profiles of the gears. By them the friction force is in direction opposite to the relative motion velocity in the contact point. So the direction of the sliding velocity $\vec{v}_{\mathrm{ta}}$ is in agreement with the direction of the friction

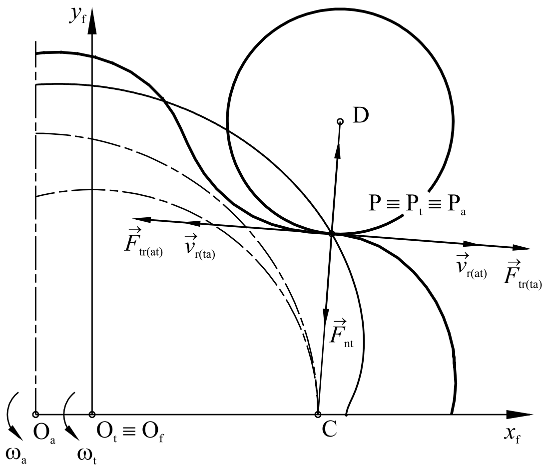


**Fig. 3**. Sliding velocities, friction forces and normal forces in the contact point of the meshing profiles

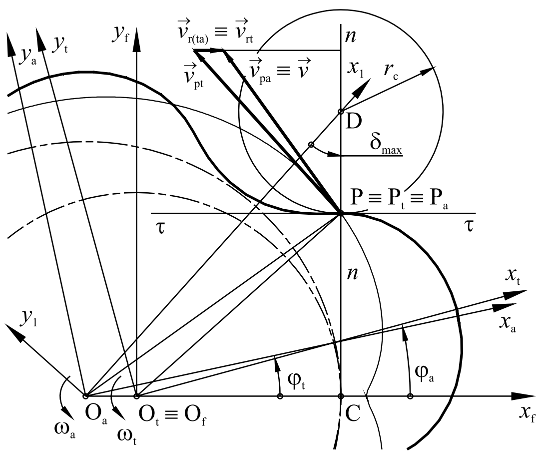


**Fig. 4**. Polygon of velocities in singular contact point

force which affects the profile of the external gear profile, but the direction of $\vec{v}_{at}$ is the same as the direction of the friction force on the trochoidal profile (Fig. 3).

It is necessary to know at the analysis of the meshing profiles sliding, except the sliding velocity in the contact point also its distribution of their change in relation to corresponding relative velocity of the contact point. Specific sliding is relation between the sliding velocity and relative velocity of the contact point of the meshing profiles[8]. After the substitution of the corresponding formulae for the velocities is obtained finally the formulae for the specific sliding on the tooth profile of the internal gear:

$$\xi_t = \frac{z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}} - c}{z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}} - c\left(1+\delta'\right)}, \tag{11}$$

and analogical for external gear:

$$\xi_a = \frac{z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}} - c}{c\delta'}. \tag{12}$$

Specific sliding is positive on the profile point, when the directions of sliding and relative velocities are in agreement, and when are not in agreement – negative.

Based on formulae (11) and (12) it can be concluded that the values of specific sliding become endless great when the values of relative velocities are equal to zero. For the distribution of specific sliding of the meshing profiles these points are singular.

At first are analysed the conditions by them $\xi_t \to \infty$, respectively, when $v_{ri}$=0. Starting of formula (3) and its making equal to zero is obtained:

$$c = \frac{z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{3}{2}}}{z+\lambda^2+\lambda\left(z+1\right)\cos\beta}. \tag{13}$$

When the value of equidistant radius is equal to the curve radius of the base epitrochoid, the obtained equation has shown that $\xi_t$ is not defined[2]. As the value of equidistant radius is choosen to be less than the minimum value of the epitrochoid curve radius, it means that the appearance of singularity for $\xi_t$ is eliminated. However, if it could be avoided the extreme great values of specific sliding, it is recommended the choice of the equidistant radius value to be considerably less than limited.

## NUMERICAL EXAMPLES

The influence of the different construction parameters on the kinematical characteristics is best illustrated through the numerical examples. In Figs 5–8 are given diagrams of the considered kinematical parameters change as a function of the angle β, for the different teeth numbers *z*, and are varied also the values of the trochoid coefficient λ and coefficient of the equidistant radius *c*.

Based on Fig. 5*a* is going out that by the same teeth numbers increase of the coefficient λ coming to enlargement of the rolling velocity $v_{rt}$ on the convex part of the profile, until on the concave part of the profile is coming to its decrease. Increase of the coefficient equidistance *c* values has opposite effect.

On the basis of Fig. 5*b* is concluded that by the same teeth numbers, increase of the coefficient λ coming to enlargement of the rolling velocity $v_{ra}$, until the enlargement of the equidistance radius has opposite effect.

Based on Fig. 6*a* it can be concluded that by the same teeth numbers, increase of the coefficient λ coming to enlargement of the sliding velocity $v_r$, until the enlargement of the equidistance radius has opposite effect. On the basis of the

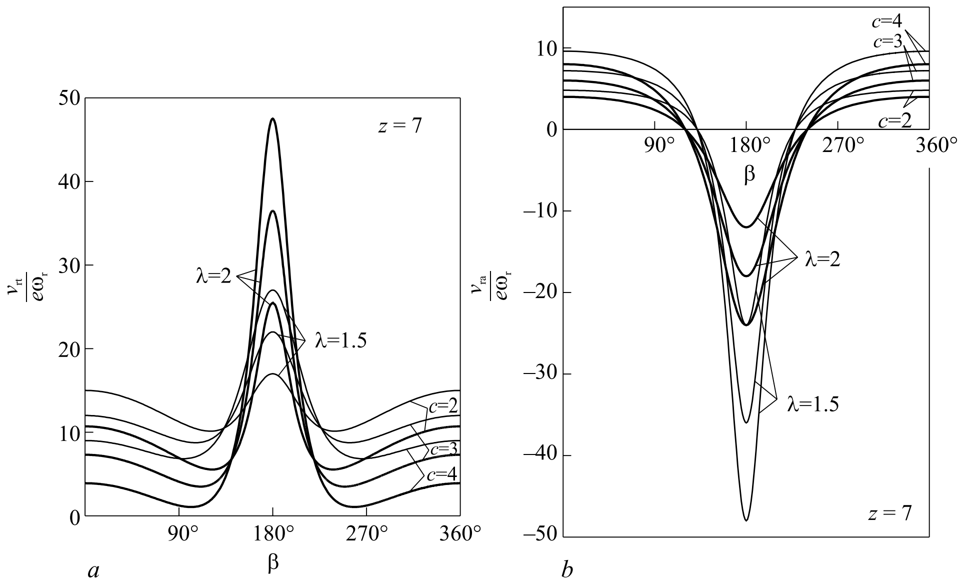


**Fig. 5**. Rolling velocity of the contact point as a function of the angle β for the teeth numbers $z=7$, at different values of the coefficient equidistance $c$ and the trochoid coefficient λ: (*a*) modified trochoidal profile, and (*b*) circular profile

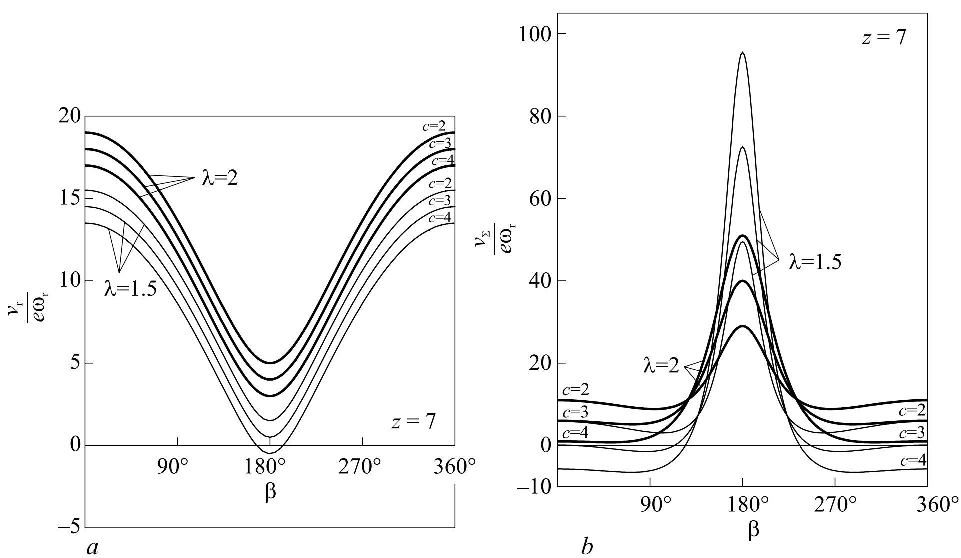


**Fig. 6**. Sliding velocity $v_r$ of the contact point of the trochoidal profile (*a*) and summary rolling velocity $v_\Sigma$ as a function of the angle β for teeth numbers $z=7$ (*b*), by different values of the coefficient equidistance $c$ and trochoid coefficient λ

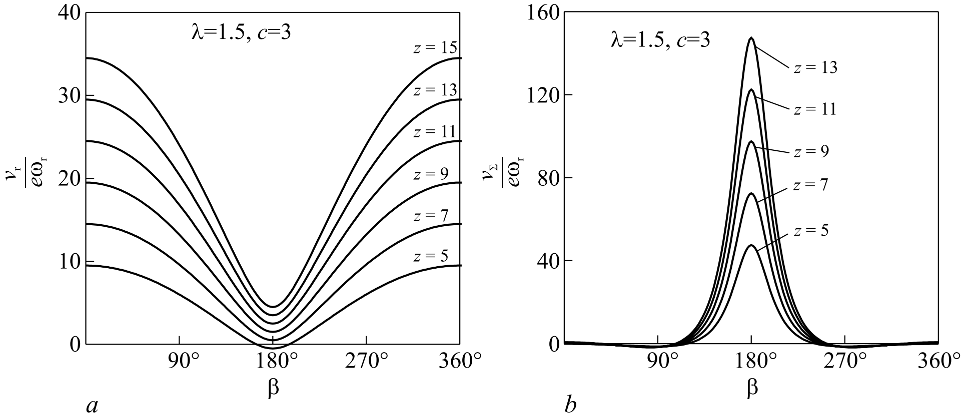


**Fig. 7**. Sliding velocity $v_r$ of the contact point of the trochoidal profile (*a*) and summary rolling velocity $v_\Sigma$ as a function of angle β for the different teeth numbers *z* (*b*), by the trochoid coefficient $\lambda = 1.5$ and coefficient equidistance $c = 3$

Fig. 6*b* is going out that by the same teeth numbers, increase of the coefficient λ coming to enlargement of the summary rolling velocity $v_\Sigma$ on the convex part of the profile, until on the concave part of the profile is coming to its decrease. Increase of the equidistance radius values has opposite effect.

Using Fig. 7*a* it can be concluded that by the same values of the coefficients λ and *c* increase of the teeth numbers has significant influence on the enlargement of the sliding velocity on the top part of the profile, until in the root part of the profile this influence is a little. In Fig. 7*b* is shown that by the same values of the coefficients λ and *c* increase of the teeth numbers has not effect on the change of the summary rolling velocity on the top part of the profile, until in the root part is coming to its abruptly increase.

Generally, can be concluded that by the choice of the geometrical parameters of the trochoidal gear pair, and aiming to decrease of the sliding velocity is recommended less teeth numbers, less values of the trochoid coefficient and more values of the equidistance radius.

In Fig. 8 is presented the distribution of the specific sliding for the pair with different parameters and based on this is established that by the same teeth number increase the coefficient λ brings to the decrease $\xi_t$ on the convex part of the profile, until on the concave part of the profile is coming to its insignificant increase. Increase of the equidistance radius values has opposite effect. Also, it can be concluded that by the same teeth number, on the top part of the circular profile, increase of the coefficient λ is resulting to enlargement $\xi_a$, until the increase of the equidistance radius values has opposite effect.

In Fig. 8*a* is shown that with the choice of the corresponding coefficient of the equidistance radius can be realised the equality of the specific sliding in the

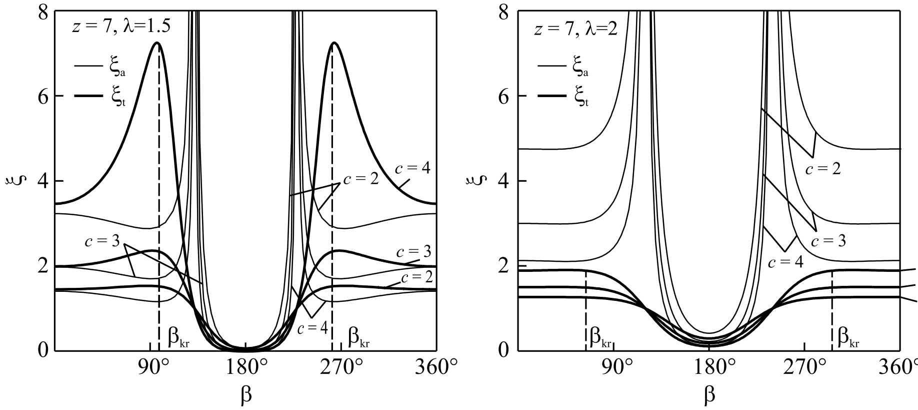


**Fig. 8**. Compared diagrams of the absolute value of the specific sliding of the profile contact point at the trochoidal gear pairs as a function of angle β for the same values of the teeth number by different values of the coefficient *c* and for different values of the trochoid coefficient λ: (*a*) $z=7$, $\lambda=1.5$ and (*b*) $z=7$, $\lambda=2$

point with the greatest sliding velocity, and it is the point on the top of the trochoid profile ($\beta=0$), as also decrease of the extreme great values of the specific sliding in area of the point with the greatest curvature of the trochoidal profile ($\beta=\beta_{kr}$).

## CONCLUSIONS

In the paper is given a detailed analysis of the kinematical parameters of the trochoidal gearing. There are defined the formulae for calculation of the sliding and rolling velocities in the contact points of the meshing profiles. It is determined the condition for the loop appearance on the contact line and the sliding velocity sign change. On the basis of the given mathematical model and corresponding graphical interpretation can be, from tribological aspect, defined the conditions for the optimum design of the considered gear pairs. Generally, it can be concluded that by the choice of the geometrical parameters of the trochoidal gear pair, and aiming to decrease the sliding velocity, is recommended less teeth numbers, and smaller values of the trochoid coefficient and higher values of the equidistance radius.

The paper also gives a detailed analysis of specific sliding at the contact points of the meshing profiles, as well as the relations for its determination. In the choice of the geometrical parameters of the conditions for appearing friction and wear of the contact gear surface specific sliding is one of the most important limiting factors. Based on mutual relations of the sliding and rolling velocities values, conclusions can be made about the changing of the friction conditions during the meshing of profiles.

On the profile part where sliding velocity is less than the summary rolling velocity the development of a fatigue pitting is expected. On the other part of the profile, the sliding velocity is greater or equal to the summary rolling velocity, so that the decrease of the lubricant layer thickness, increase of temperature in contact and risk of the scoring appearance are expected on this part.

On the basis of the obtained formulae can be concluded that the values of the specific sliding become endless great when the values of the relative velocities are equal to zero. These points are singular for the distribution of the specific sliding of the meshing profiles. On the profile of the internal gear the specific sliding has not singular points, so that on the profile of the external gear exists a point with endless great specific sliding.

Except that, in this paper is shown that with the corresponding choice of the geometrical parameters can be realised the equality of the specific sliding in the points with the greatest sliding velocity, in regard to, from the kinematical aspect, ensured the uniform teeth wear of the meshing gears in the process of the meshing profiles.

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