THE 7TH INTERNATIONAL CONFERENCE RESEARCH AND DEVELOPMENT OF MECHANICAL ELEMENTS AND SYSTEMS

THE EFFECT OF GEOMETRY ON THE STRESS DISTRIBUTION OF CROSS SHAFT

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Abstract: *In this paper will be represented how changes in geometry can affect on size of the stress change cross shaft of cardan joint. The task of the cardan joint is mechanical transmission of power and motion between shafts which are changing position of the axis in the process of exploitation or are placed at certain angles. Cross shaft is one of the most important parts of the cardan joint. In most cases the lifetime of cardan joint depend from the cross shaft. The critical stress of cross shaft will be calculated using the analytical method and been tested by numerical method. To the numerical method used as reliable for further change the geometry cross shaft. We can see how small changes in geometry of cross shaft can lead to a significant reduction on critical stress. Increasing radius in the transition and basis sleeve of the cross shaft reduces the critical stress at the root up to 40%. It will be the comparisons of the results in order to come to the best of the geometric shape of a cross shaft which has the lowest critical stress.*

Key words: *cross shaft, the critical stress, numerical method, geometric shape.*

1. INTRODUCTION

Cardan joints are used to connect misaligned shafts that are intersecting. They transmit rotational motion from one shaft to another. Cardan joints, also referred to as either universal joints or Hooke joints have been used for many years in mechanical devices such as automobiles and aircraft. These applications required small joint angles. Cardan joint can be used under high speed, large operating angle and heavy load conditions [1, 2, 3].
In the development of agricultural and transport mechanical engineering occurs rapid development cardan mechanisms and their increasing use. For mobile transportation and agricultural machines, which are in motion are subjected to significant shaking and change the position of some of its shafts, it was necessary for mounting such a mechanism does not react to changes in the position of the shaft axis, and thereby maintain good exploitation properties of the machine.
Cardan joints are used in: transport and agricultural machinery, cars and locomotives, radioelectronic devices, machine-tools, drills and pumps the oil industry, control mechanisms aircraft and helicopters, wood industry, textile industry and so on [4].
Correct placement of cardan mechanisms, in terms of structural composition, the possibility of greater freedom a designer in solving mutual arrangement shaft transmitters. But the use of cardan joint in some cases, lead emerging large dynamic loads (hydraulic transmissions and so on).

1.1. Cross shaft

Cross shaft is one of the most important parts of the cardan joint. In most cases the size and lifetime depend on the cardan joint from the cross shaft.
Depending on the type of cardan mechanism, it is possible to use different variants of structures cross shaft (Figure 1) [4].

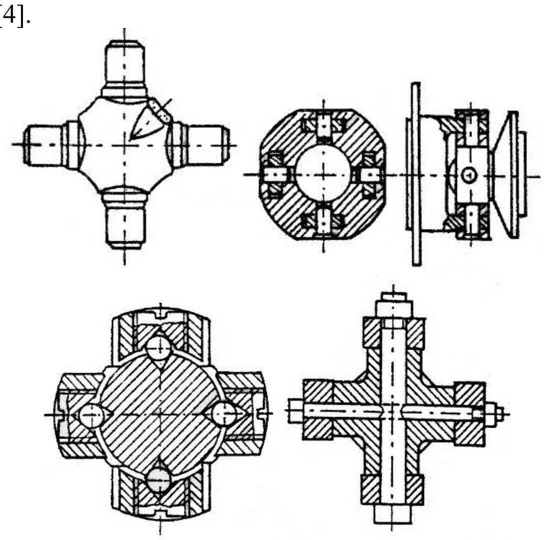

Fig.1. Construction variants of cross shaft [4]

Legs cross shaft are flex to:
• bending axis in the plane cross shaft,
• bending in the plane of the fork joint from the moment of friction in the telescope and
• shear forces.
Cross shaft sleeves in addition to the above suffer even more stress on the surface pressure, if they are supported

in sliding bearings or on contact pressure if they are supported in the roller bearings, for example needle bearings.
If you are due to inaccurate fabrication and assembly radial clearances in needle bearings larger than axial (between the sleeve and the bottom of a cup) head sleeve can rely on the bottom of the cup and be exposed to the surface pressure.
Maximum bending increases from the middle of cross shaft to the inlet. Load distribution along the rolling elements is the increase in part from sleeve.

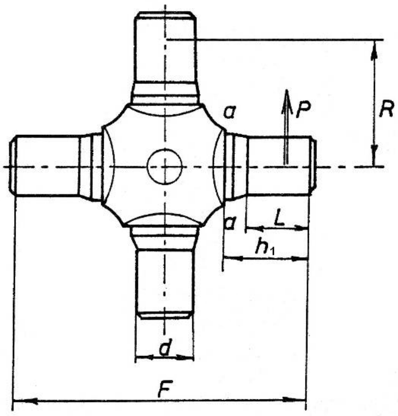

Fig.2. Cross shaft [5]

Bending stresses in the base sleeve cross shaft (critical section a-a), if you ignore the center hole for lubrication, can be determined as follows (Figure 2) [5]:

$$\sigma_s = \frac{Pl}{W_s}, \tag{1.1}$$

where is:

$$l = h_1 - \frac{L}{2}. \tag{1.2}$$

The maximum force on to the inlet of cross shaft is:

$$P = \frac{M_{u2\max}}{2R} = \frac{M_{u2\max}}{F-L}, \tag{1.3}$$

and the resistant bending moment on sleeve:

$$W_s = \frac{\pi d^3}{32}, \tag{1.4}$$

so that the bending stress:

$$\sigma_s = \frac{32 M_{u2\max}\left(h_1 - \frac{L}{2}\right)}{(F-L)\pi d^3}. \tag{1.5}$$

In equations meaning labels is:
h_1 - distance between the front sleeve and budget section,
L - length of needle bearings,
F - the distance between the front of cross shaft and
d - sleeve diameter.
Shear stresses on cross shaft can be determined by equation:

$$\tau = \frac{4P}{\pi d^2}. \tag{1.6}$$

Taking into account that the on can make cross shaft of different types of steel, some experiences show that it is good that the bending stresses do not exceed (15-30)·10^7 Pa for passenger cars and (15-25)·10^7 Pa for commercial vehicles.
Analysis of the actual stress distribution and behavior of cross shaft of cardan joints in operation shows that the initial cracks as the beginning of the destruction usually occurs in the zone below the nipple hole. Strength on cross shaft can increase construction solutions in which the central lubrication hole sets at the head of branches, as well as increasing the radius of curves between the sleeve and the middle part on cross shaft.
For the localization of the harmful effects of concentrated contact stresses, which inevitably leads to overload and plastic deformation of the contact elements, it is necessary to:

- gap adjustment: backwater - needles - the cup from 10-40 μm and
- production of tapered sleeve on the length of needles in proportion depending on the size of the radial working gap.

Tests have shown that reduced levels of radial clearance between the cross shaft and needle bearing from 0.080 to 0.018 mm allows, depending on the load circuit, increasing the number of rolling elements in contact from 28% to 90%.
The range of contact pressures of 2000 - 3000 MPa can be 20 to 30% increase torque transmission.
Noteworthy are constructions of cross shaft with pressed sleeves made of materials with high hardness. Sleeves are placed on the branch. Such construction allow the application of progressive methods of machining and substantially extend the lifetime of the cross shaft.
Contact strength of cross shaft is increased by applying sulfating and other forms of reinforcement and thermomechanical treatment on work surfaces. On lifetime influence and roughness, where the optimal roughness achieved with superfinishing.

2. ANALYTICAL CALCULATION CROSS SHAFT OF CARDAN JOINT

For cross shaft of cardan joint that was in exploitation was carried out analytical and numerical calculation of with actual measures from the model. And then performed varying the model and the stress state of numerical methods in order to reach the best variant with the lowest stress [6]. The basic data for calculation are given in the Table 1.

Table 1. Basic data

Name	Value
Entry power	P_U=10 kW
Entry number of rotation	n_1=1500 min^{-1}
Slope	α_{12}=30^o
Shear modulus	G=0,8·10^{11} Pa
Dimensions of cross shaft	d=11,5 mm
	F=40 mm
	h_1=10,5 mm
	L=9 mm

- Torsion moment on the drive shaft

$$M_{U1} = \frac{P_U}{\pi \cdot n_1 \cdot 30} = 63{,}662 \text{ Nm}$$

- Maximum and minimum transmission ratio

$$i_{12\max} = \frac{1}{\cos \alpha_{12}} = 1{,}155$$

$$i_{12\min} = \cos \alpha_{12} = 0{,}866$$

- Angular velocity on the drive shaft

$$\omega_1 = \frac{n_1 \cdot \pi}{30} = 157{,}08 \text{ s}^{-1}$$

- The maximum and minimum angular velocity of the output shaft

$$\omega_{2\max} = \omega_1 \cdot i_{12\max} = 181{,}38 \text{ s}^{-1}$$

$$\omega_{2\min} = \omega_1 \cdot i_{12\min} = 136{,}035 \text{ s}^{-1}$$

- Maximum and minimum torque twisting on the output shaft

$$M_{u\max 2} = \frac{M_{U1}}{i_{12\min}} = 73{,}511 \text{ Nm}$$

$$M_{u\min 2} = \frac{M_{U1}}{i_{12\max}} = 55{,}133 \text{ Nm}$$

- Constant torque component on the output shaft

$$M_{2k} = M_{U1} \cdot \frac{1 + \cos \alpha_{12}}{2 \cos \alpha_{12}} = 68{,}586 \text{ Nm}$$

- Variable torque component on the output shaft

$$M_{2p} = M_{U1} \cdot \frac{\sin^2 \alpha_{12}}{2 \cos \alpha_{12}} = 9{,}189 \text{ Nm}$$

- Maximum and minimum torque torsion on the output shaft

$$M_{u2\max} = M_{2k} + M_{2p} = 77{,}775 \text{ Nm}$$

$$M_{u2\min} = M_{2k} - M_{2p} = 59{,}397 \text{ Nm}$$

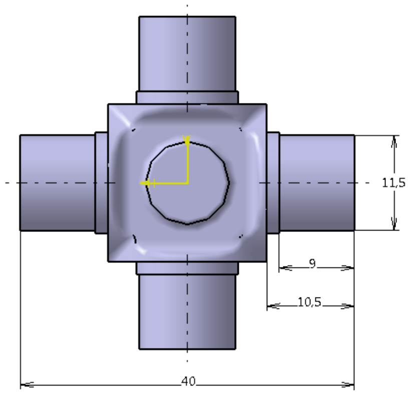

Fig.3. Cross shaft

- Distance l

$$l = h_1 - \frac{L}{2} = 0{,}006 \text{ m}$$

- The maximum force on the branch of cross shaft

$$P_k = \frac{M_{u2\max}}{F - L} = 2{,}509 \text{ kN}$$

- Resistant bending moment

$$W_{sk} = \frac{\pi \cdot d^3}{32} = 1{,}439 \cdot 10^{-7} \text{ m}^3$$

- The bending stress

$$\sigma_{sk} = \frac{P_k \cdot l}{W_{sk}} = 1{,}008 \cdot 10^8 \text{ Pa}$$

- The shear stress

$$\tau_k = \frac{4 \cdot P_k}{\pi \cdot d^2} = 2{,}415 \cdot 10^7 \text{ Pa}$$

- The equivalent stress

$$\sigma_{ek} = \sqrt{\sigma_{sk}^2 + 3 \cdot \tau_k^2} = 1{,}092 \cdot 10^8 \text{ Pa}$$

The stress is below the allowable stress which is σ_d=15·108 Pa.

3. STRUCTURAL FINITE ELEMENT ANALYSIS OF CROSS SHAFT

Finite element analysis is a powerful tool in the field of engineering. Initially, finite element analysis was used in aerospace structural engineering. The technique has since been applied to nearly every engineering discipline from fluid dynamics to electromagnetics.

The difficulty in analysis of stress and strain in structural engineering depends on the structure involved. As the structure grows in complexity, so does the analysis. Many of the more commonly used structures in engineering have simplified calculations to approximate stress and strain. However, these calculations often provide solutions only for the maximum stress and strain at certain points in the structure. Furthermore, these calculations are usually only applicable given specific conditions applied to the structure [7].

A finite element stress analysis is carried out at the failure region to determine the stress distribution and possible design improvement [8].

Engineering analysis and design require users to make several assumptions, typically at different levels.

One of these levels is the choice of the underlying mathematical model of the system. Another level is the description of the model parameters. Assumptions are usually made to facilitate processing the analysis and design, which result in that the nature of engineering computation is conditioned by a priori assumptions. In a deterministic analysis, the geometry, loads, and material properties are assumed to have specific values [9].

Structural finite element analysis, made in CATIA® software, used to check analytical stress values obtained for the cross shaft cardan's joints and in order to be used as relevant for the effect of changes in the shape of the size of maximum stress. We can see how the changes on the model affect the maximum stress.

Cross shaft is an element that is symmetrical and has four branches arranged at an angle of 90 degrees to each other. On each of these branches operate the same force that is transmitted through the bearings. As there are four of the same forces that symmetrical load on cross shaft, so the numerics can also extract and observe only one-quarter cross shaft, which is loaded by one force. Figure 4 presents a model pin cross and Figure 5 is allocated its quarter to be used in finite element analysis.

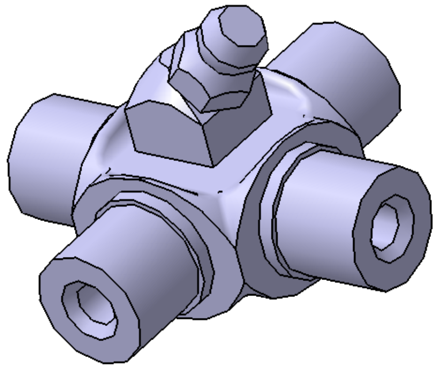

Fig.4. Cross shaft

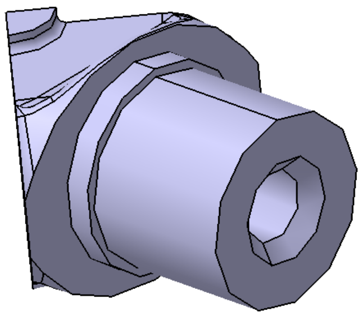

Fig.5. Quarter of cross shaft

When calculating cross shaft started from the simplified model in order to get as close to the results and then carried out the finite element analysis of the actual models, and the modified models to reduce stress. In preparing calculation and in order to seek optimal solutions that have arisen in the several varieties. Each cross shaft is considered in the same conditions, weighed the same kind of force in the same way, which has a value of 2059 N with a mesh size on the model of 1.5 x 0.5 mm and local fragmentation of mesh size in the same place from 1 mm.

1. Simplified model that differs from the that are left open for the passage of lubricant, because the analytical calculation was made for the full pin, so it is justifiable to use this pin. Image of this model is shown in Figure 6, a stress distribution for it in Figure 7.

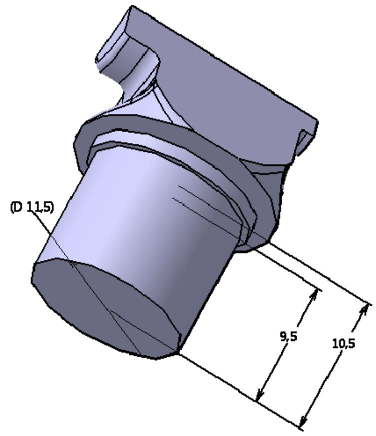

Fig.6. A quarter of the full model cross shaft

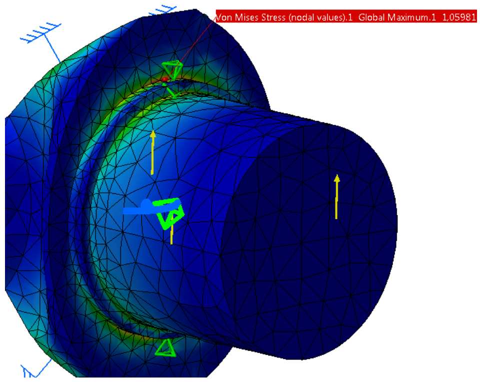

Fig.7. Stress of the quarter cross shaft

The maximum stress that occurs at the root is 105.981 MPa, which is about 3% different from the equivalent stress obtained by the analytical method, which has a value of 109.2 MPa. Given that the deviations from the values of low stress further in the numerical method can be reliably used to compare values obtained by modifying the model. So you can see how small changes in the model affect the size of maximum stress.

2. The actual cross shaft is different from the previous corresponding cross shaft simplified analytical model because in it there are openings for the passage of lubricant. The model was made based on the actual cross shaft is shown in Figure 8 the stress distribution with maximum values in Figure 9.

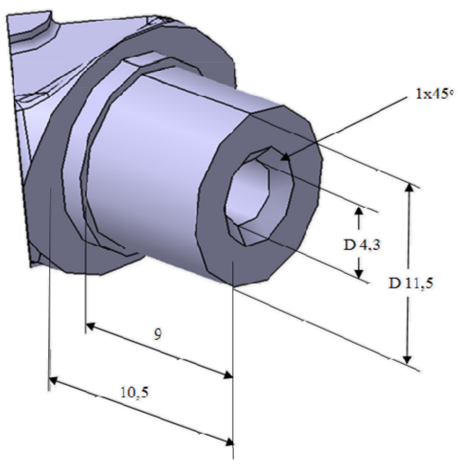

Fig.8. A quarter model cross shaft made by a real model

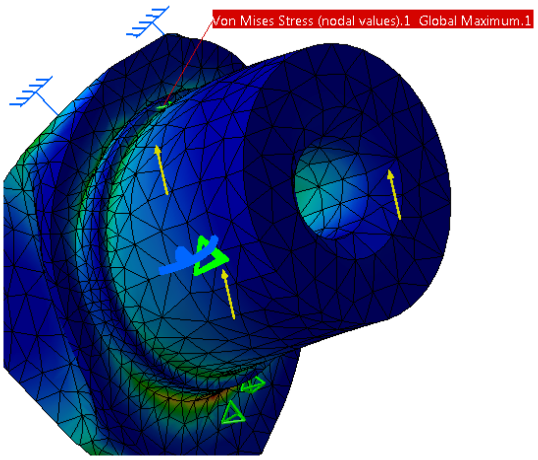

Fig.9. Stress of the quarter at the actual cross shaft

By comparing the results with the previous case notes that the values are very close, stress is very little changed from 105.981 to 105.032 MPa, which is less than 1%. While this value is less of stresses obtained analytically by about 4%. So we can conclude that opens a lubricant have a negligible impact on sizing stress.

3. How does the actual cross shaft curves, in order to effect a reduction in the size of the concentrated stress was introduced curvature of 0.5 mm at the turn and at the root of shaft. Figure 10 presents a model derived from a change in Figure 11 is a given stress distribution.

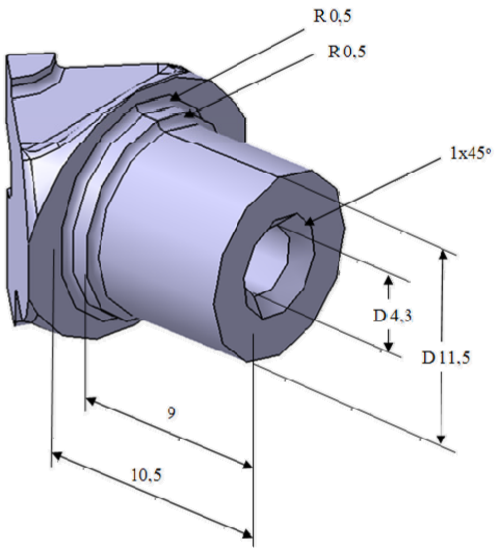

Fig.10. A quarter model cross shaft made with a change of R at the root (case 3)

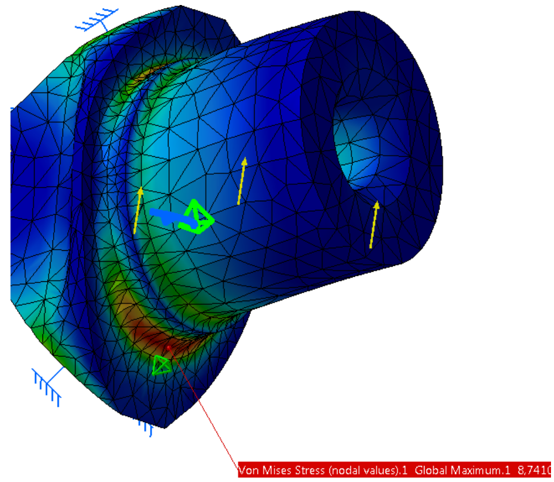

Fig.11. Stress of the quarter cross shaft with a change of curvature

At the root of the maximum stress occurs which has a value of 87.411 MPa, which is almost 17% smaller than the stress on the previous model and up to 20% less of stresses obtained analytically, which means that the curve of the reduced stress concentration at the critical section.

4. In this case, was detained at the turn radius of 0.5 mm and increased radius at the root of 1 mm. Modified model is shown in Figure 12 while the stress state of such a model is given in Figure 13.

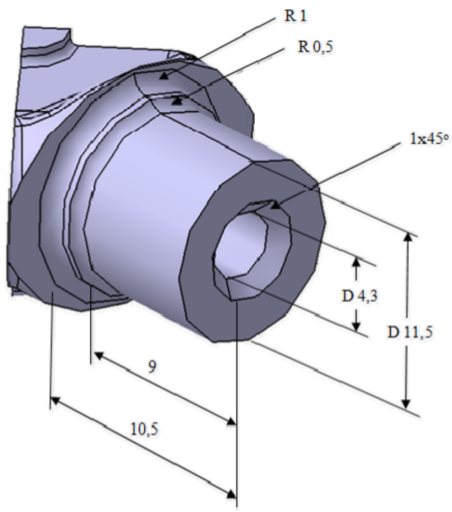

Fig.12. A quarter model cross shaft made with a change of R at the root (case 4)

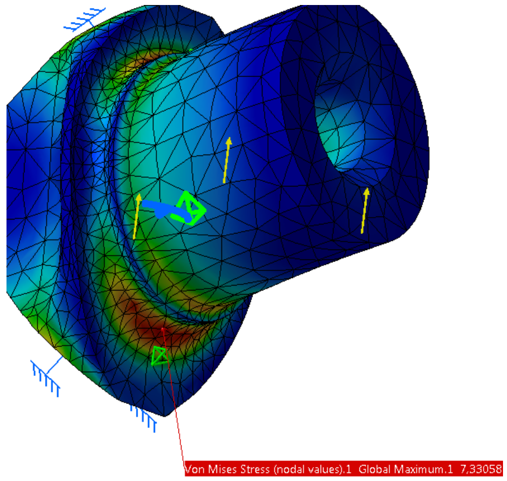

Fig.13. Stress of the quarter cross shaft at the modified model (case 4)

The maximum stress that occurs at the base has a value of 73.306 MPa, which would mean that the loss in reduced stress concentration at that point by 16% compared to the previous case and about 33% compared to analytically derived stress.

5. In this example, both the radius of the same size are 1 mm. In Figure 14 presents a model with a changing radius curves in Figure 15 is a data file stress for the model.

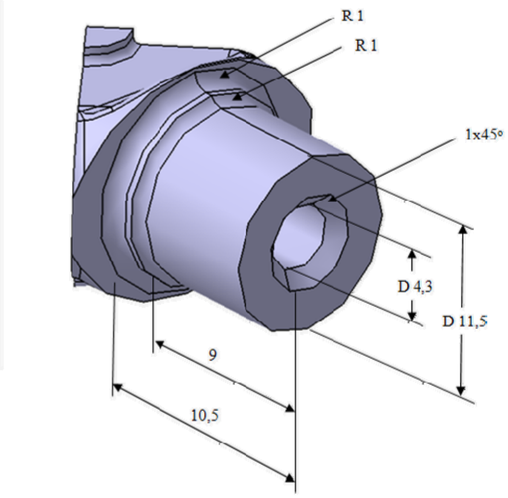

Fig.14. A quarter model cross shaft made with a change of R at the root (case 5)

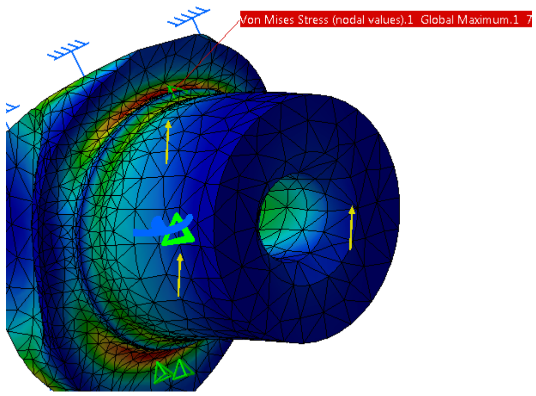

Fig.15. Stress of the quarter cross shaft (case 5)

Maximum stress occurs at the base, the area of stress concentration with the size of 74.094 MPa, which means that the stress increased by 1% compared to the previous case. So that this variant cross shaft worse than the previous one because it occurs at a higher stress.

6. If we continue with increasing radius, and the root of the radius increases from 1 to 1.5 mm, there is a change of stress state in the model and reduces the stress concentration there. At the same time the other dimensions remain unchanged from the previous example. Figure 16 gives the changed model and Figure 17 is a stress distribution.

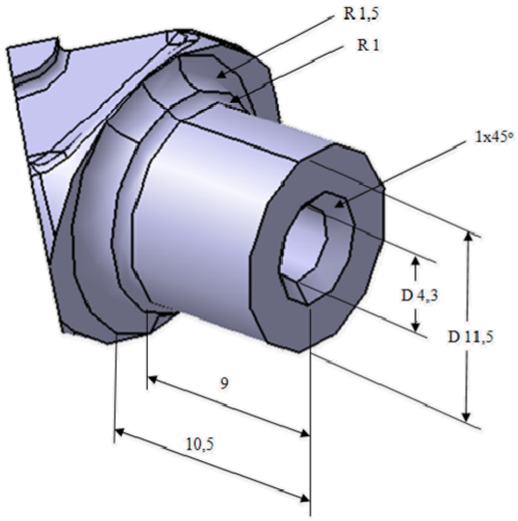

Fig.16. A quarter model cross shaft made with a change of R at the root (case 6)

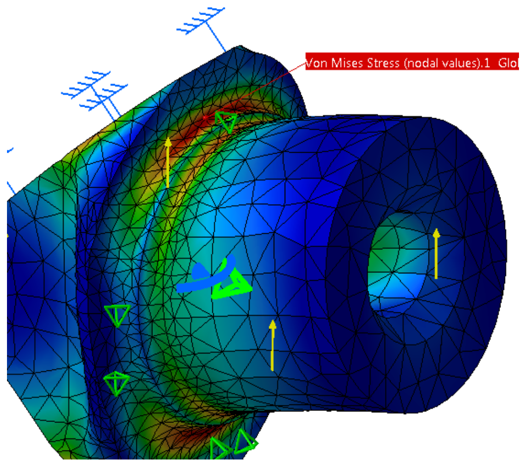

Fig.17. Stress distribution at the quarter cross shaft (case 6)

Introduced by changing the radius of curves is obtained to the maximum stress of 61.353 MPa that occurs at the root of cross shaft, a decrease of 17% compared to the previous example, with respect to the stress obtained by analytic reduction is 44%.

7. If it continues to increase at the transition curves, from 1 to 1.5 mm, change to the maximum stress value. Changed model will look like in Figure 18 and the stress state of this model is given in Figure 19.

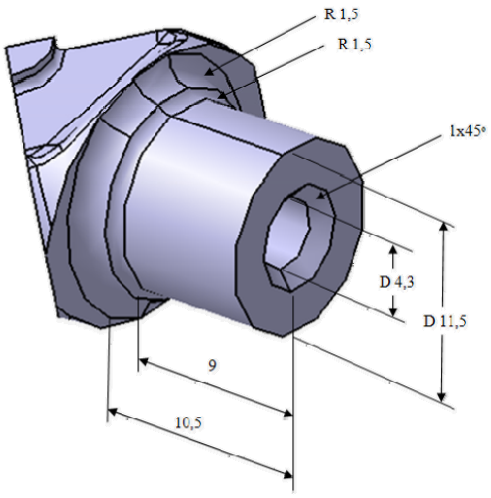

Fig.18. A quarter model cross shaft made with a change of R at the root (case 7)

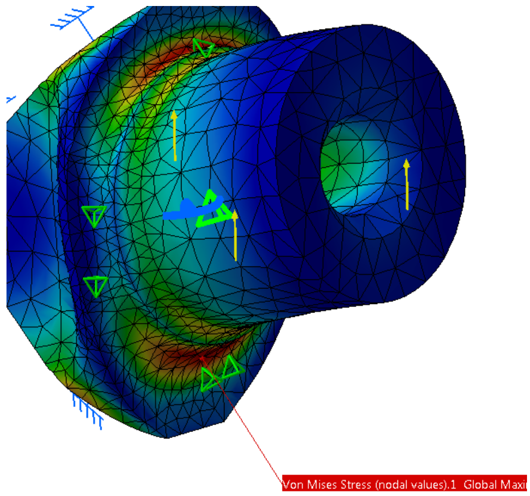

Fig.19. Stress state of the quarter cross shaft at a real model (case 7)

Introduced by changing the radius of curves is obtained to the maximum stress of 62.497 MPa that occurs at the root of cross shaft but on the opposite side from the previous example and has a higher value by 2.5% compared to the same. So that such a solution worse than the last.
So it's not optimal exercise increased more rounded at the turn because it leads to increased stress, and the cheapest variant of the sixth, which is rounded at the turn of 1 mm and rounded at the base of 1.5 mm.

4. ANALYSIS OF RESULTS

Since the analysis cross shafts cardan's joint can choose the cheapest option which has the lowest stresses.
The following Table 2 along given maximal stress variations, at the cross shaft. Case with lowest stress is marked and chosen as most favorable.
This is the case 6, where the radius of the root radius of 1.5 mm and the gate 1 mm. In this case the stress reduced by 43.82% compared to the stress derived by the analytical method.

Table 2. Maximal stress with the percentage difference at the cross shaft

Analytically calculate the stress: 109,2 MPa			
No	Maximum stress	Stress difference from the previous model [%]	Stress difference from the analytically calculated [%]
1	105,98 MPa		2,95
2	105,03 MPa	0,90	3,82
3	87,41 MPa	16,78	19,95
4	73,31 MPa	16,14	32,87
5	74,09 MPa	-1,07	32,15
6	61,35 MPa	17,20	43,82
7	62,95 MPa	-2,60	42,36

5. CONCLUSION

Based on analysis conducted the cross shaft cardan's joint, it can be concluded that small changes in shape can lead to large changes in stress state of the investigated part.
Stress concentration can be reduced curves. Increasing curves reduces stress concentration to a marginal extent, followed by an increase in curves increases and stress.
The results received by analytical calculation can not be reliably used as authoritative, because they are made for a general simplified model. With more complex shape and keeping the same dimensions relevant for calculation, the stress can be changed by more than 50%, which is by analytical calculation can not be seen. So have to use the numerical calculation, that the calculation takes into account the shape, so that any change in the shape influence the change in stress.
By the examining the cross shaft, it was concluded that the grooves in the transition and base sleeve can reduce stresses up to 40%. The studies indicated that the cheapest option when the two different radius curves, with the smaller radius at the turn of a larger underlying sleeve.
To verify the results it is desirable to use some other software in which the numerical method for the same models loaded in the same way to get more reliable results.
Further research in this field will be tested how change, not only curves, but also changes form the central part of the cross shaft affects on the change in stresses.

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