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STRESS AND STRAIN STATE OF CYCLOID DISC

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Abstract: *This paper deals with stress and strain state analysis for cycloid disk, as a vital element at cycloidal speed reducer, for a case when machining tolerances exist. Three cases of the most critical meshing have been analysed: single, double and triple meshing. Values of forces between the rollers of the stationary central gear and cycloid gear were analytically calculated. Stress and strain state analysis were numerically realised by application of FEM. The whole range of models with different cases of load distribution has been made. Obtained results are showed in a form of figures, diagrams and tables. Finally, conclusions were made, based on realised analysis.*

Keywords: cycloidal speed reducer, cycloid disc, stress state, strain state

1. INTRODUCTION

The most important application of the cycloid profile gears (cycloid gears) is their use at cycloidal speed reducers. Due to a range of good characteristics they posses, and firstly due to a big gear ration and low losses, cycloidal speed reducers are very much used within modern industrial machines. Considering the fact that they posses very compact design, they can be readily applied for devices with space limitations. Model of the single-stage cycloidal speed reducer is shown in Figure 1.

The most important element of the cycloidal speed reducer is certainly cycloid gear. As a teeth profile of the cycloid gear, equidistant of the shortened epitrochoid is the most frequently used. Cycloidal gear teeth are meshed with rollers of the stationary central disk. For theoretical case, when machining tolerances are not considered, half

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of cycloidal gear teeth participate at the load transmission process. The most frequently, for each transmission rate, two identical cycloid disks are used, rotated 180º relative to each other. This way, good dynamical equilibrium of cycloidal speed reducer elements is obtained and also the ability to accept even the significantly high short-time overloads (up to 500%) without failures.

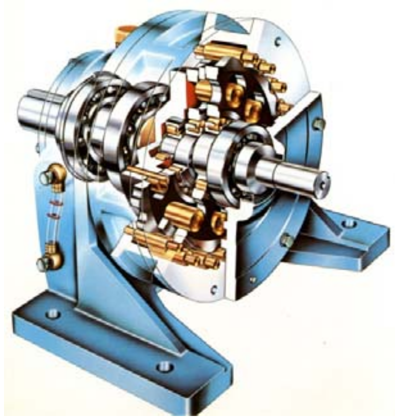

Fig. 1. Single-stage cycloidal speed reducer

The way of cycloidal gearing generation is defined by Kudrijavcev [1], and after that, in more details by Lehmann [2]. Litvin and Feng [3] derived parametric equations of the trochoid and its equidistant. Blanche and Yang [4] defined inter-dependencies between machining tolerances, drive parameters (namely, gear ratio, pitch diameter and normalized tooth heights) and drive performance (namely, backlash and torque ripple), by using equations in vector form.

Analysis of loads that appear at cycloidal speed reducer, for a case when machining tolerances do not exist, is represented in papers [5] and [6]. Papers [7] and [8] describe the procedure for force calculation of cycloidal speed reducer elements, for the case of modified tooth profile of cycloid disk. Yunhong, Changlin and Liping have taken into account friction force during load analysis [9], whilst in paper [10], experimental analysis of efficiency coefficient of cycloidal speed reducer is done. Chmurawa and John [11] have done analysis of stress state and strain state of cycloid gears when machining tolerances do not exist and when all teeth of cycloid disks are in contact with appropriate housing rollers. Static and dynamic forces act upon elements of the cycloidal speed reducer. Dynamical models of involute gear pair and cycloidal speed reducer are represented in papers [12].

This paper deals with analysis of stress state and strain state of cycloid gears for the case when machining tolerances exist in the presence of static forces.

2. CYCLOID DISC LOADS

Cycloid disk is the most significant element of the cycloidal speed reducer due to its complex geometry, but also due to a complex stress state occurring within it. In order to realise analysis of the stress and strain state, it is necessary to define forces which act upon it. Cycloid disk in contact with housing rollers and output rollers, [5] is shown in Figure 2.

Forces on cycloid disc are:

F_E – bearing reaction,

F_{Ni} – force between housing roller i and cycloid disc,

F_{Kj} – force between output roller *j* and cycloid disc,
T_1 – input torque.

The following equations can be expressed, based on Figure 2:

$$T_1 = F_E \cdot e \cdot \cos(\beta + \varepsilon) \tag{1}$$

$$T_1 = \frac{r_i}{z} \cdot \sum_{j=1}^{q} F_{Kj} \cdot \sin\left(\beta_j + \beta\right) \tag{2}$$

$$\sum_{i=1}^{p} F_{Ni} \cdot \cos\alpha_i - \sum_{j=1}^{q} F_{Kj} \cdot \cos\beta - F_E \cdot \sin\varepsilon = 0 \tag{3}$$

$$F_E \cdot \cos\varepsilon - \sum_{i=1}^{p} F_{Ni} \cdot \sin\alpha_i + \sum_{j=1}^{q} F_{Kj} \cdot \sin\beta = 0 \tag{4}$$

$$\sum_{i=1}^{p} F_{Ni} \cdot l_i - \sum_{j=1}^{q} F_{Kj} \cdot r_i \cdot \sin(\beta_j + \beta) = 0 \tag{5}$$

where:
e – eccentricity,
β – swivel angle of the input shaft,
ε – angle betce F_E and eccentricity e direction,
r_i – radius of output rollers pitch circle,
z – number of teeth of cycloid disc (gearing ratio of the cycloidal speed reducer),
β_j – angular position of the output roller *j*,
α_i – angle which force F_{Ni} makes with vertical,
l_i – lever arm of force F_{Ni},
p – number of housing rollers that carry the load,
q - number of output rollers that carry the load.

Values α_I and l_i are calculated according to Figure 2, based on the following expressions:

$$\alpha_i = \operatorname{arctg} \frac{\sin\beta + \frac{r}{r_2} \cdot \sin\gamma_i}{\cos\beta - \frac{r}{r_2} \cdot \cos\gamma_i} \tag{6}$$

$$l_i = r_1 \cdot \sin(\alpha_i - \beta) \tag{7}$$

Angle γ_i (angular position of the housing rollers) is calculated based on the following expression:

$$\gamma_i = \frac{360 \cdot \left(2 \cdot i - 1\right)}{2 \cdot \left(z + 1\right)} \tag{8}$$

where:
r - radius of housing rollers pitch circle,
r_1 - base circle radius of the cycloid disc,
r_2 - base circle radius of the housing rollers.

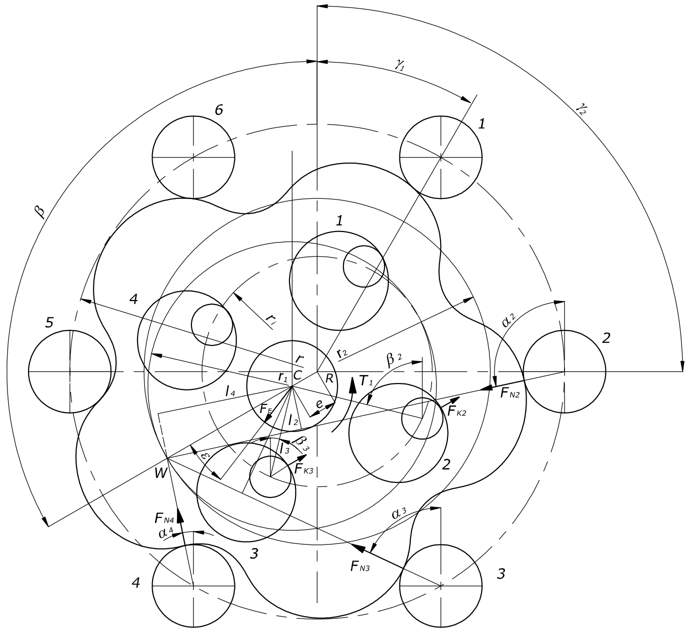

Fig. 2. Cycloid disc in contact with housing rollers and output rollers

Forces F_{Ni} and F_{Kj} are proportional to their respective distances from the centre of rotation:

$$\frac{F_{\mathrm{Ni}}}{l_{\mathrm{i}}} = const. \tag{9}$$

$$\frac{F_{\mathrm{Kj}}}{\sin(\beta_{\mathrm{j}} + \beta)} = const. \tag{10}$$

Only for ideal (theoretical) case all cycloid disk teeth are in contact with appropriate rollers and half of them carry load. In reality, cycloidal speed reducer has machining tolerances due to which number of teeth in contact is lower than in ideal case, that is, the load per one tooth is increased.

This paper deals with three the most critical cases (single, double and triple meshing). For observed single-stage cycloidal speed reducer (P = 5,5 kW, n = 1500 min $^{-1}$, z = 11 , e = 4 mm, r_1 = 44 mm , r_2 = 48 mm , r = 72 mm) values of forces F_{Ni} are calculated, based on expressions (1) – (10) and results are given in Table 1.

Table 1. *Values of forces between housing rollers and cycloid disc*

Meshing:	F_{N1}, *N*	F_{N2}, *N*	F_{N3}, *N*
Single	*4628,5*	-	-
Double	*2211,1*	*2312*	-
Triple	*1969,3*	*2059,1*	*998,3*

3. ANALYSIS OF STRESS – STRAIN STATE OF CYCLOID DISC USING FEM

Analysis of stress and strain state of cycloid disk is realised using FEM. For this purpose, the whole range of numerical models has been made. Cycloid disc is considered to be deformable elastic body. Three the most critical cases (single, double and triple meshing) are analysed, for the value of the swivel angle of the input shaft β = 20º . It is considered that at one instant two output rollers are in contact with cycloid disc, for all numerical models. Supports have been set up at these points. Bearing reaction is decomposed into nine components and supports have been set up also at points of these components. External loads are forces F_{Ni}. Problem is observed as being planar. Quadrilateral two-dimensional isoparametric finite elements have been used. Cycloid disc model consists of 9227 finite elements and 9753 nodes. Steel 30CrMoV9 (WNr 1.7707) was selected as a cycloid disc material, with the following characteristics:

- Yield stress: R_{eH} = 700 MPa,
- Tensile strength: R_m = 1100 MPa,
- Modulus of elasticity: $E = 2{,}1 \cdot 10^5$ MPa,
- Poisson coefficient: $\mu = 0{,}3$.

Two from 6 analysed numerical models with loads and limitations presented, are shown in Figures 3 and 4.

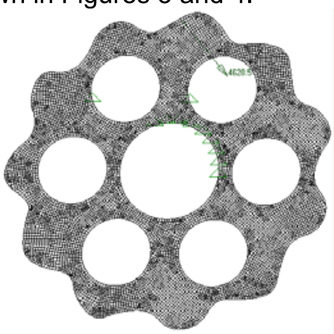

Fig. 3. Numerical model of cycloidal disc for the case of single meshing when force F_{N1} acts in one node

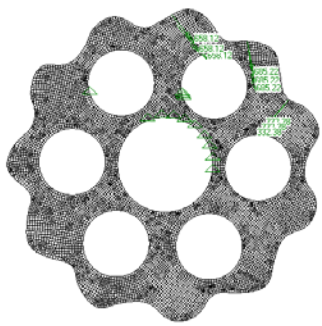

Fig. 4. Numerical model of cycloidal disc for the case of triple meshing when each of the forces F_{N1}, F_{N2} and F_{N3} is decomposed into three components

Beside analyzing single, double and triple meshing, stress and strain state of cycloid disc are analysed for cases when forces between housing rollers and cycloid disc are each decomposed into the three components, where at components of forces act upon the neighbouring nodes.

4. NUMERICAL ANALYSIS RESULTS

Stress and strain state analysis is realised by FEMAP software. Specifically, modeling, pre- and post-processing is done by FEMAP, while the calculation itself is done by MSC NASTRAN software. Stress and strain state for different models is represented in Figures 5, 6, 7, 8 and 9.

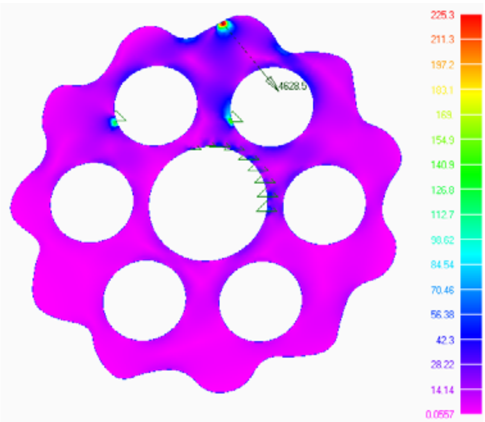

Fig. 5. Von-Mises stress distribution for the case of single meshing when force F_{N1} acts in one node

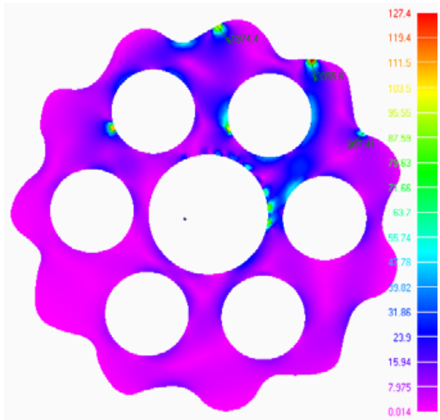

Fig. 6. Von-Mises stress distribution for the case of triple meshing when each of the forces F_{N1}, F_{N2} and F_{N3} act in one node

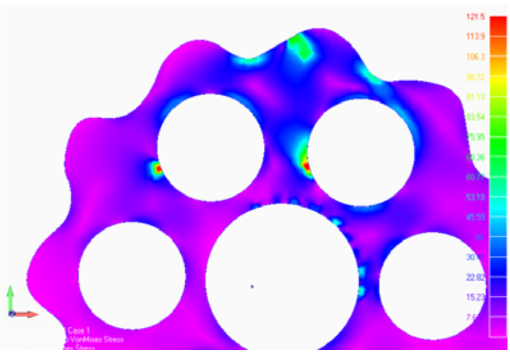

Fig. 7. Von-Mises stress distribution for the case of single meshing when force F_{N1} is decomposed into three components

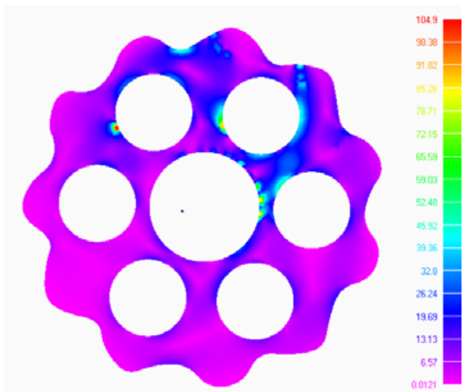

Fig. 8. Von-Mises stress distribution for the case of triple meshing when each of the forces F_{N1}, F_{N2} and F_{N3} is decomposed into three components

Maximum values of Von Mises stress and strain are depending on single, double or triple meshing and also on the forces between housing rollers and cycloid disc acting at one or three nodes. Diagrams are shown in figures 9 and 10.

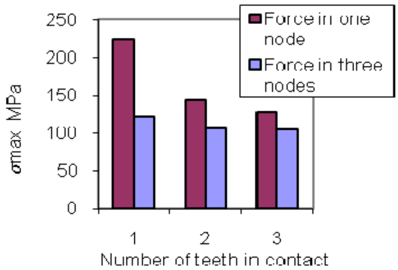

Fig. 9. Comparative analysis of Von Mises stress values when forces between housing rollers and cycloid disc act at one or three nodes

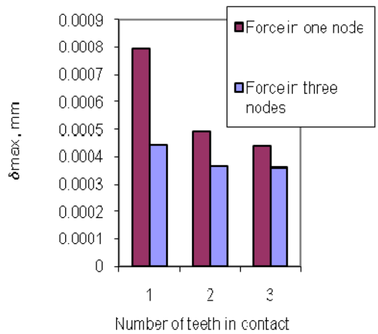

Fig. 10. Comparative analysis of maximum strain values when forces between housing rollers and cycloid disc act at one or three nodes

Values of stress and strain are the largest in case of single meshing and when total force acts at one node, what can be observed on diagrams in Figures 9 and 10. In that case: σ_{max} = 225,3 MPa and δ_{max} = 0,000795 mm. The lowest values of stress and strain are in case of triple meshing and when each of the forces F_{N1}, F_{N2} and F_{N3} is decomposed into three components. In that case: σ_{max} = 104,9 MPa and δ_{max} = 0,000360 mm. Obviously, values are almost doubled compared to each other, what certainly should be taken into account during design and manufacturing of the cycloidal speed reducer.

5. CONCLUSIONS

As with all machines and devices, there exist machining tolerances at cycloidal speed reducer. As a consequence to it, all housing rollers are not all simultaneously in contact with cycloid disc and do not all carry load. Three the most critical cases have been analysed in the paper, from aspects of stress and strain values. The cases of single, double and triple meshing have been analysed, when the total force between housing roller and cycloid disc acts at one node and also when that force is divided into three components and act at three neighbouring nodes.

Based on realised stress-strain analysis using FEM, the following conclusions can be made:

- The most unfavorable case, from aspect of stress and strain values, is the case of the single meshing when total force F_{Ni} acts at one node and the most favorable case is the triple meshing when each of the forces F_{N1}, F_{N2} and F_{N3} is decomposed into three components which act upon neighbouring nodes (stress values are in this case more than two times lower);
- Maximum stress and strain values, for numerical models where total force F_{Ni} acts at one node, are located exactly at these nodes (Figures 5 and 6);
- When forces F_{Ni} are decomposed into components, maximum stress and strain values are located in the contact zone of output rollers and cycloid disc - openings at the body of the cycloid disc (Figures 7 and 8);
- Even in the case of the most unfavorable single meshing, maximum stress and strain values are within the limits that provide reliable work of cycloid disks during the foreseen working life, what is extremely good recommendation for even more extensive use of cycloidal speed reducers.

LITERATURE

[1] Kudrijavcev, V.N. (1966). Planetary Gear Train (in Russian), *Mechanical Engineering,* Lenjingrad.
[2] Lehmann, M. (1976). Calculation and Measurement of Forces Acting on Cycloid Speed Reducer (in German), *PhD Thesis,* Technical University Munich, Germany.
[3] Litvin, F., Feng P. (1996). Computerized Design and Generation of Cycloidal Gearings, *Mechanism and Machine Theory*, vol. 31, no. 7, p. 891-911.
[4] Yang, D.C.H., Blanche, J.G. (1990). Design and Application Guidelines for Cycloid Drives With Machining Tolerances, *Mechanism and Machine Theory*, vol. 25, no. 5, p. 487-501.
[5] Malhotra, S.K., Parameswaran, N M.A. (1983). Analysis of a Cycloid Speed Reducer, *Mechanism and Machine Theory*, vol.18, no. 6, p. 491-499.
[6] Blagojević, M. (2008). Stress and Strain State of Cyclo Speed Reducer`s Elements Under Dynamic Loads (in Serbian), *PhD Thesis,* Faculty of Mechanical Engineering, Kragujevac, Serbia.
[7] Chmurawa, M., Lokiec, A. (2001). Distribution of Loads in Cycloidal Planetary Gear (CYCLO) Including Modification of Equidistant, *16th European ADAMS User Conference,* Berchtesgaden, Germany.
[8] Lixing, L., Xin, L., Weidong, H., Yuanmei, Q. (1999). Profile Modification and Accurate Force Analysis on Cycloid Drive, *4th World Congress on Gearing and Power Transmission*, Paris, France, vol. 3, p. 1141-1146.
[9] Yunhong, M., Changlin, W., Liping, L. (2007). Mathematical Modeling of the Transmission Performance of 2K-H Pin Cycloid Planetary Mechanism, *Mechanism and Machine Theory*, vol. 42, no. 7, p. 776-790.
[10] Davoli, P., Gorla, C., Chiozzi, F. (2007). Theoretical and Experimental Analysis of a Cycloidal Speed Reducer, *10th ASME International Power Transmission and Gearing Conference*, Las Vegas, USA.
[11] Chmurawa, M., John, A. (2002). Numerical Analysis of Forces, Stress and Strain in Planetary Wheel of Cycloidal Gear Using FEM, *Journal of Mechanical Engineering*, vol. 53, no. 2, p. 77-92.
[12] Blagojević, M., Nikolić, V., Marjanović, N., Veljović, Lj. (2009). Analysis of Cycloid Drive Dynamic Behavior, *Scientific Technical Review*, vol. LIX, no. 1, p. 52-56.