



STRESS AND STRAIN STATE OF SINGLE – STAGE CYCLOIDAL SPEED REDUCER

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Abstract: This paper deals with analysis of stress-strain state for some elements of single-stage cycloidal speed-reducer when machining tolerances exist. Due to existence of machining tolerances, teeth of cycloid disc are not all at simultaneous contact with appropriate rollers of the central stationary gear and do not carry load. In order to efficiently do analysis, one single-stage cycloidal speed-reducer has been designed.

Analysis of cycloid disc stress-strain state is realised using FEMAP software package. Cycloid disc is considered to be elastic deformable body. The cases when one, two or three pairs of teeth are in contact simultaneously have been analysed. Problem has been considered as being planar.

Analysis of the stress and strain that occur in the contact of the stationary central gear and the pins on which rollers are positioned and which conjugate with cycloid disc teeth is realised. Analysis has been done using CATIA software package, with appropriate 3-dimensional numerical models made for this purpose. Numerical analysis of stress and strain state for eccentric was realised, too.

Based on analysis of obtained results, conclusion can be made that from aspect of stress distribution, the most unfavourable case is the theoretical case when one pair of teeth is in contact, what was expected. Even in a case of the most unfavourable single meshing, maximum stress and strain values are within the limits that provide reliable work of cycloid disc during the foreseen working life, what is extremely good recommendation for even more extensive use of cycloidal speed reducer.

Key words: cycloidal speed reducer, cycloid disc, stress state

1. INTRODUCTION

Cycloidal speed reducers belong the group of planetary gears. Their inventor is German engineer Lorenz Braren (1931). Because of very wide area of application, production of cycloidal speed reducer has growing character. Their main characteristics are: big transmission ratio, low losses, compact design, reliable work,... Model of the cycloidal speed reducer is shown in Figure 1.

The most important element of the cycloidal speed reducer is cycloid disc which teeth profile is equidistant of the shortened epitrochoid. In theoretical case, when machining tolerances are not considered, half of cycloid disc teeth participate at the load transmission process.

The basic information about cycloidal gearing are presented by Kudrijavcev [1] and by Lehmann [2]. Litvin and Feng [3] developed parametric equations for equidistant of trochoid. Calculation of forces which acting on cycloidal speed reducer elements, when machining tolerances don't exist, is defined in papers [4, 5, 6]. Dynamic loads are dominant at cycloidal speed reducer. Their dynamic behaviour is presented in papers [7, 8]. Chmurawa and Lixing [9, 10] described the distribution of loads at cycloid disc with modified tooth profile.

Experimental analysis of cycloidal speed reducer is presented in paper [11].

This paper deals with analysis of stress and strain state of cycloidal speed reducer elements when machining tolerances exist.

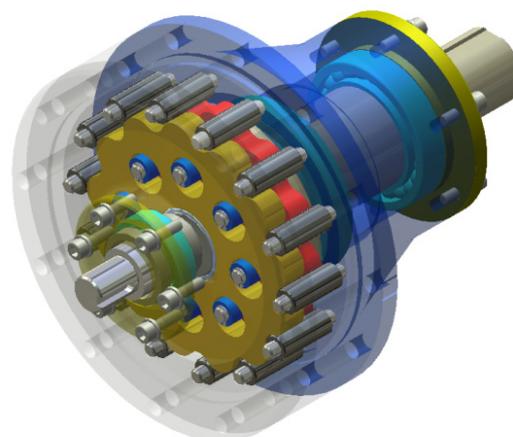


Fig. 1: Model of cycloidal speed reducer

2. LOADS ON CYCLOIDAL SPEED REDUCER ELEMENTS

Cycloid disc is the most important element of the cycloidal speed reducer due to its complex geometry and complex stress and strain state. In the first step it is necessary to define forces which act upon it. Cycloid disc with contact elements (housing rollers and output rollers) is shown in Figure 2.

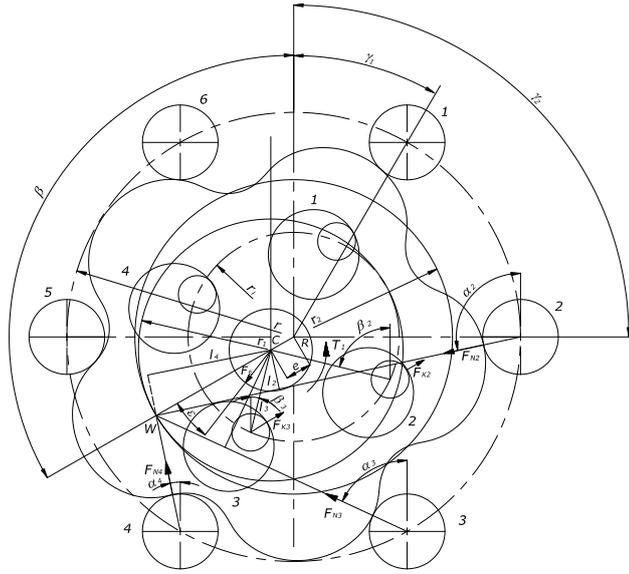


Fig.2: Cycloid disc in contact with housing rollers and output rollers

Forces on cycloid disc are:

- F_E – bearing reaction,
- F_{Ni} – force between housing roller i and cycloid disc,
- F_{Kj} – force between output roller j and cycloid disc,
- T_1 – input torque.

The following equations can be expressed, based on Figure 2:

$$T_1 = F_E e \cos(\beta + \varepsilon) \quad (1)$$

$$T_1 = \frac{r_1}{z} \sum_{j=1}^q F_{Kj} \sin(\beta_j + \beta) \quad (2)$$

$$\sum_{i=1}^p F_{Ni} \cos \alpha_i - \sum_{j=1}^q F_{Kj} \cos \beta - F_E \sin \varepsilon = 0 \quad (3)$$

$$F_E \cos \varepsilon - \sum_{i=1}^p F_{Ni} \sin \alpha_i + \sum_{j=1}^q F_{Kj} \sin \beta = 0 \quad (4)$$

$$\sum_{i=1}^p F_{Ni} l_i - \sum_{j=1}^q F_{Kj} r_1 \sin(\beta_j + \beta) = 0 \quad (5)$$

where:

- e – eccentricity,
- β – swivel angle of the input shaft,

ε – angle between the force F_E and eccentricity direction,

r_1 – radius of output rollers pitch circle,

z – number of teeth of cycloid disk (gearing ratio of the cycloidal speed reducer),

β_j – angular position of the output roller – j ,

α_i – angle which force F_{Ni} makes with vertical,

l_i – lever arm of force F_{Ni} ,

p – number of the housing rollers that carry the load,

q – number of the output rollers that carry the load.

Values α_i and l_i are calculated according to Figure 2, based on the following expressions:

$$\alpha_i = \arctg \frac{\sin \beta + \frac{r}{r_2} \sin \gamma_i}{\cos \beta - \frac{r}{r_2} \cos \gamma_i} \quad (6)$$

$$l_i = r_1 \sin(\alpha_i - \beta) \quad (7)$$

Angle γ_i (angular position of the housing rollers) is calculated based on the following expression:

$$\gamma_i = \frac{360(2i-1)}{2(z+1)} \quad (8)$$

r – radius of housing rollers pitch circle,

r_1 – base circle radius of the cycloid disc,

r_2 – base circle radius of the housing rollers.

Forces F_{Ni} and F_{Kj} are proportional to their respective distances from the centre of rotation:

$$\frac{F_{Ni}}{l_i} = const. \quad (9)$$

$$\frac{F_{Kj}}{\sin(\beta_j + \beta)} = const. \quad (10)$$

Only for ideal (theoretical) case all cycloid disk teeth are in contact with appropriate rollers and half of them carry load. In reality, cycloidal speed reducer has machining tolerances due to which number of teeth in contact is lower than in ideal case, that is, the load per one tooth is increased.

This paper deals with three the most critical cases (single-, double- and triple meshing). For observed single-stage cycloidal speed reducer ($P = 5,5 \text{ kW}$, $n = 1500 \text{ min}^{-1}$, $z = 11$, $e = 4 \text{ mm}$, $r_1 = 44 \text{ mm}$, $r_2 = 48 \text{ mm}$, $r = 72 \text{ mm}$) values of forces F_{Ni} are calculated, based on expressions (1) – (10) and results are given in Table 1.

Table 1: Values of forces between housing rollers and cycloid disc

Force	Single meshing	Double meshing	Triple meshing
F_{N1} , N	4630	2210	1970
F_{N2} , N	-	2310	2060
F_{N3} , N	-	-	1000

3. ANALYSIS OF STRESS-STRAIN STATE CYCLOID SPEED REDUCER ELEMENTS

Analysis of stress and strain state of cycloidal speed reducer elements (cycloid disk, stationary central gear and eccentric) is realised using FEM. For this purpose, the whole range of different numerical models has been made. All these elements are considered to be deformable elastic bodies. Numerical model of cycloid disc is presented on Figure 3.

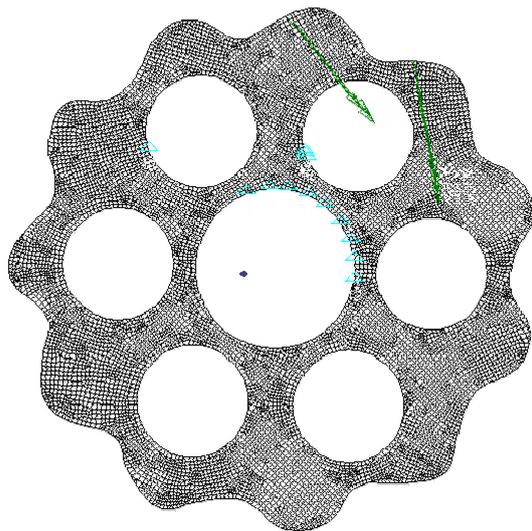


Fig.3: Numerical model of cycloid disc for double meshing

Three the most critical cases (single-, double- and triple meshing) are analyzed. It is considered that at one instant two output rollers are in contact with cycloid disc, for all numerical models. Supports have been set up at these points. Bearing reaction is decomposed into nine components and supports have been set up also at points of these components. External loads are forces F_{Ni} . Problem is observed as being planar. Quadrilateral two-dimensional isoparametric finite elements have been used. Cycloid disk model consists of 9227 finite elements and 9753 nodes. Steel 30CrMoV9 was selected as a cycloid disk material, with the following characteristics:

- Yield stress: $R_{eH} = 700$ MPa ,
- Tensile strength: $R_m = 1100$ MPa ,
- Modulus of elasticity: $E = 2,1 \cdot 10^5$ MPa ,

-Poisson coefficient: $\mu = 0,3$.

Stress and strain state analysis of cycloid disc is realized by FEMAP software. Stress and strain state for some different cases are presented in Figures 4, 5, 6 and 7.

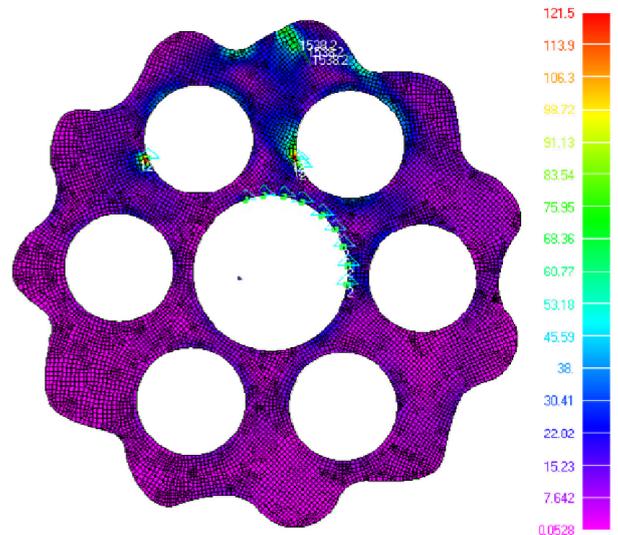


Fig.4: Von-Mises stress distribution of cycloid disc for single meshing

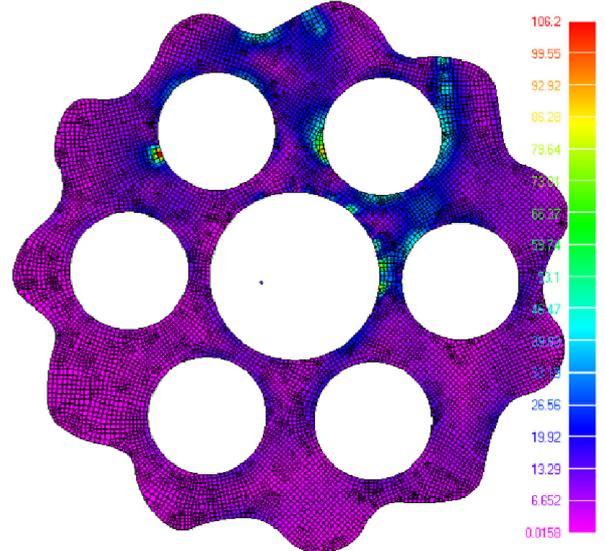


Fig.5: Von-Mises stress distribution of cycloid disc for double meshing

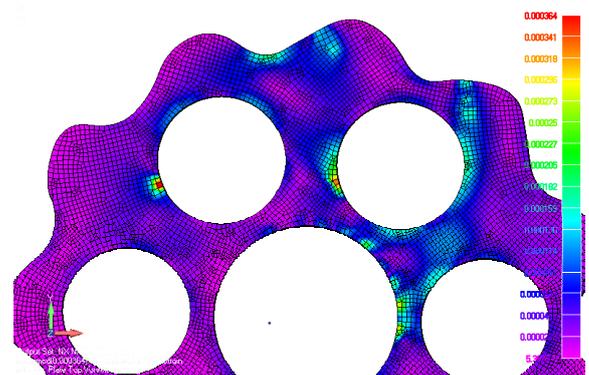


Fig.6: Strain state of cycloid disc for double meshing

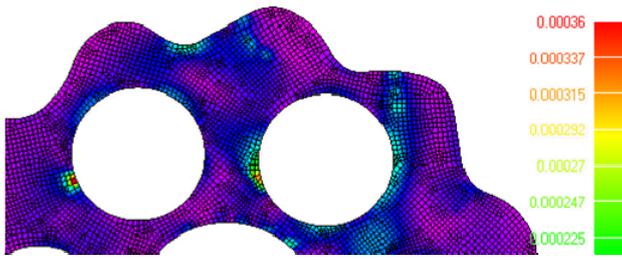


Fig.7: Strain state of cycloid disc for triple meshing

Maximum values of Von-Misses stress and strain are depending on single-, double- or triple meshing. This values are presented in Figure 8 and 9.

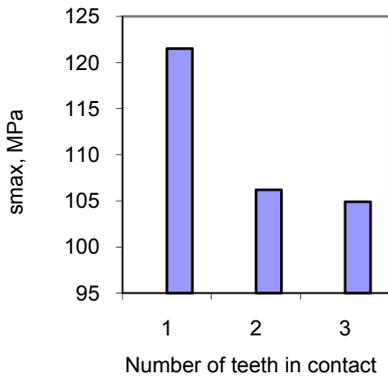


Fig.8: Comparative analysis of Von-Misses stress values for cycloid disc

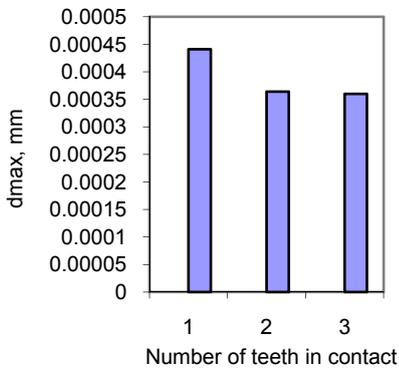


Fig.9: Comparative analysis of maximum strain values for cycloid disc

Values of stress and strain are the largest in case of single meshing. In that case: $\sigma_{max} = 121,5\text{MPa}$ and $\delta_{max} = 0,000441\text{mm}$. For triple meshing there are: $\sigma_{max} = 104,9\text{MPa}$ and $\delta_{max} = 0,00036\text{mm}$.

Model of stationary central gear is presented in Figure 10.

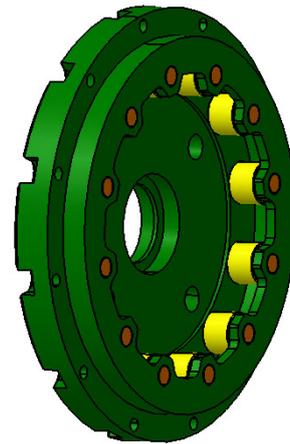


Fig.10: 3D model of stationary central gear

Numerical analysis of stationary central gear was done in software CATIA. 3D curvilinear tetrahedral finite elements were used. Numerical model of this element is presented in Figure 11.

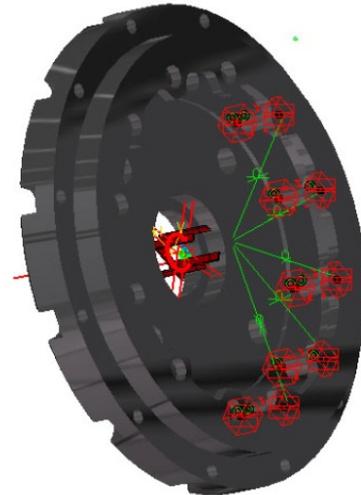


Fig.11: Numerical model of stationary central gear

In Figures 12 and 13 are presented areas with maximum values of contact stress and strain.

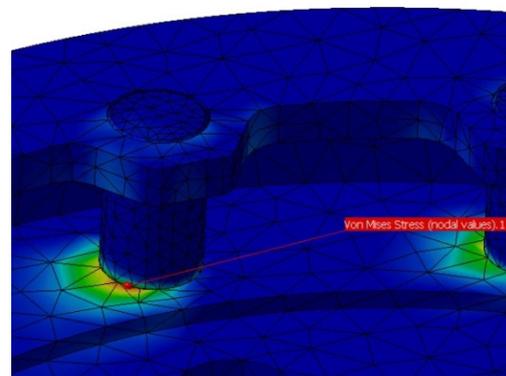


Fig.12: Areas with maximum values of contact stress at stationary central gear

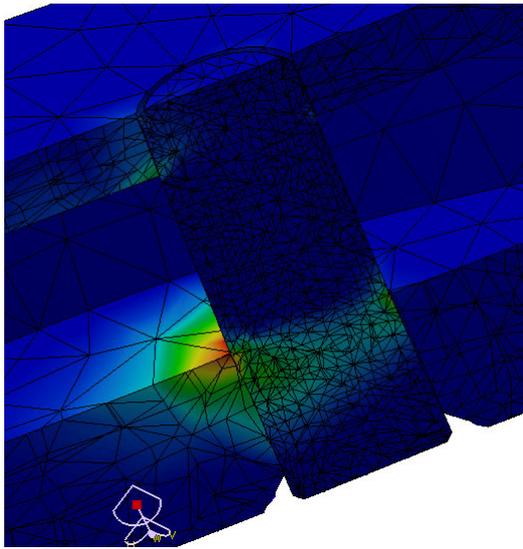


Fig.13: Areas with maximum values of strains at stationary central gear

3D model (CATIA) of eccentric with needle bearings is presented in Figure 14.

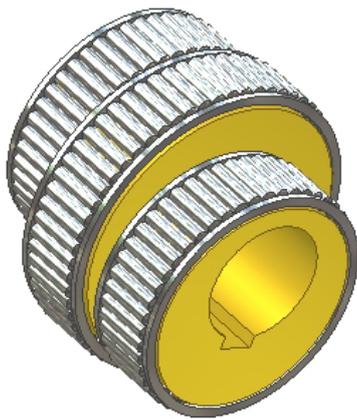


Fig.14: 3D model of eccentric

Numerical model of eccentric with external loads and supports is presented on Figure 15.

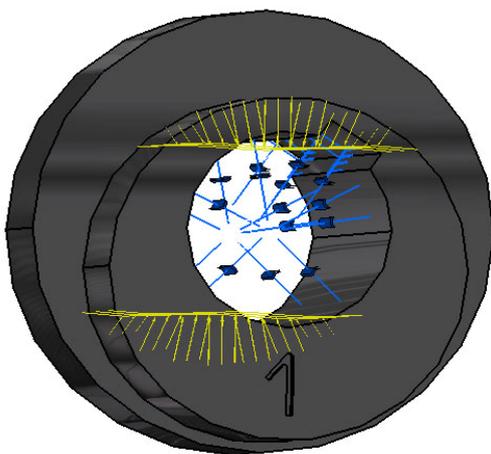


Fig.15: Numerical model of eccentric

Area of maximum displacement at eccentric is presented in Figure 16.

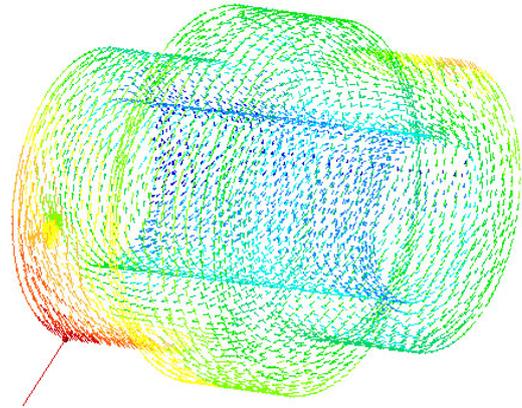


Fig.16: Area of maximum strains at eccentric

In Table 2 the maximum values of stress and strain at analyzed elements of cycloidal speed reducer for the most critical cases are presented.

Table 2: Maximum values of Von-Misses stress and strain at analyzed elements of cycloidal speed reducer

Element of cycloidal speed reducer	Max Von-Misses stress, MPa	Max strain, mm
Cycloid disc (single meshing)	121,5	0,000441
Cycloid disc (double meshing)	106,2	0,000364
Cycloid disc (triple meshing)	104,9	0,00036
Stationary central gear	86	0,00026
Eccentric	95	0,000302

CONCLUSION

As with all machines and devices, there exist machining tolerances at cycloidal speed reducer. As a consequence to it, all housing rollers are not all simultaneously in contact with cycloid disc and do not all carry load. Three the most critical cases for cycloid disc have been analyzed in the paper, from aspects of stress and strain values. Problem is observed as being planar using software FEMAP.

There are numerical analysis for stationary central gear and for eccentric, too. Analysis for this elements was done in software CATIA. 3D finite elements were used.

Based on realized stress-strain analysis using FEM, the following conclusions can be made:

- The most unfavorable case for cycloid disc, from aspect of stress and strain values, is the case of the single meshing and the most favorable case is the triple meshing;

-Even in the case of the most unfavorable single meshing, maximum stress and strain values are within the limits that provide reliable work of cycloid disks during the foreseen working life;

-Values of contact stress and strains for stationary central gear and for eccentric are very close to values of Von-Misses stress and strains of cycloid disc. It means that all vital elements are very uniformly loaded and this is very important advantage of the single stage cycloidal speed reducer.

ACKNOWLEDGMENT

The results of this paper are realized through the National project TR – 35037 financially supported by the Ministry of Science of the Republic of Serbia (coordinator of project Prof. dr R. Slavkovic).

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