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SHAPE VARIATIONS INFLUENCE ON LOAD CAPACITY OF CARDAN JOINT CROSS SHAFT

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Abstract: *Design of the machine elements' optimal shape due to stress concentration is one of the major problems in the design of power transmitters. Analyze of shape variations influence on load capacity of Cardan joint' cross shaft, as one of the elements of mechanical power transmitter is presented in this paper. Functional tasks of power transmitters with Cardan shafts are to transmit power and motion between shafts with variable mutual position of axis or with axis which are forming certain angle. Load capacity of Cardan joint is highly influenced by design parameters which define form of cross shaft. Maximal stress act at the zone of cross shaft' section changing from central part to the branches and represent critical stress for the whole power transmitter. Critical stress at cross shaft was determined, in this paper, by analytical approach and verified by finite element method. By the variations of geometrical parameters and repetitions of numerical calculations of stresses with invariant load conditions, the optimal fillet in critical section of cross shaft was created in order to obtain minimal stress concentration.*

Key words: *cross shaft, critical stress limit, design*

1. INTRODUCTION

The main function of power transmitter with Cardan shafts is to transmit power and torque between the elements of car's power group which are on certain distance and that are not rigidly connected. They can be used between elements without coincide in axes or with variation in relative positions. The Cardan shafts are used as part of power transmitter on vehicles which have engine in the front and rear wheel drive. They transmit the torque from the shaft to the main pinion gear and further to planetary transmitters. The Cardan shaft with two joints is usually used. This type of shaft provides flexibility of positions to the angle of 30° between of axes. This characteristic is important due to fact that driving wheels have continual up and down movement in order to reduce the influence of impacts from the road to the car body, so the need for flexible transmission of power and torque is essential. From this aspects, the usage of Cardan shafts in power transmitters of cars and trucks, building and agricultural machinery. The Cardan shafts are used to deliver power and torque to the additional equipment of tractors and other machines when position flexibility of power transmission is essential [1].

The identification of Cardan shafts and its elements are based on nominal power and torque from catalogs of producers. But, Cardan shaft, chosen by this method, do not always satisfied the requirements for reliability and duration of exploitation period. In most cases, the cause of early replacement of those elements are not production process, but design of whole power transmitter.

The causes of failures and design of power transmitters with Cardan shafts are analyzed by many researches. Hummel and Chassapis [2] researched on the design of the universal joints. They have given some suggestions on the configuration design and optimization of universal joints with manufacturing tolerances [3]. Bayrakceken et al. [4] performed the fracture analysis of a universal joint yoke and a drive shaft of an automobile power transmission system. Spectroscopic analyses, metallographic analyses and hardness measurements are carried out for each part. For the determination of stress conditions at the failed section, stress analyses are also carried out by the finite element method. The reference [5] considered modification of design of Cardan shaft in order to avoid failures during exploitation period. The modifications of designs are analyzed by finite elements methods and the best modification of design by decrease of dimensions of input Cardan joint yoke is identificated.

For the rational design, safety and reliability evaluations of machines' elements it is necessary to determine the stress levels and its distributions in the critical zones. The stress level and its distributions depend on load characteristics so as on the shape of the machines' elements. At the zones with variation in shape and dimensions of cross section the stresses are irregularly distributed and the maximal stresses are far greater than nominal stresses. Besides that, the multiple stress concentrations as the consequence of multiple stresses concentrators influence are induced [6]. For the aim of reducing the stress concentration the design and technological procedures are done. By the increase of fillets at critical zones, the stress concentrations can be significantly reduced. But, the possibilities of this procedure are limited due to interferes with axial support of bearings. In this paper the procedure of identification of optimal combination of shape and dimensions of shape transition zones from the aspects of maximal stresses reductions is shown.

2. STRESS STATE AT CRITICAL ZONE OF CROSS SHAFT OF CARDAN JOINT

Cross shaft is one of the most important elements of power transmitters with Cardan joints. In most cases its characteristics are significant factor for dimensions and properties so as duration of exploitation period and reliability of whole power transmitter. The presented facts implicate that identification of optimal dimensions and shape at critical zone of cross shaft is essential for avoiding of failure.

The branches of cross shafts are loaded to: flexion in the cross shaft plane, flexion in the plain of Cardan joint joke of connection shaft form the torque due to friction in telescopic element and shear due to forces. Maximal bending momentum of forces is rising from the central zones to the branches.

In the study presented in this paper, the stress limit at the cross shaft is calculated by analytical method, and then, obtained result is proven by numerical method. Finite elements method is used to analyze the stress distribution in critical zone. On the bases of the obtained results the optimization of cross shaft design was done.

2.1. Analytical calculation of stresses

The calculation of stresses by analytic method is done for the design of cross shaft, presented at Fig. 1. The dimensions of considered model are limited by construction requirements. The basic properties of considered power transmitter with Cardan joint are: power P=25 kW, number of rotation n=2000 min^{-1}, distance between top of branches and critical cross section h_1=21.5 mm, the length of bearing zone $h_2 = 17.5$ mm, distance between two top sides of opposite branches L=70 mm, diameter of branches d=18 mm, diameter of hole for supply of lubricant d_1=4 mm, the angle between input and output Cardan joint jokes α_{12max}=18.78° [7].

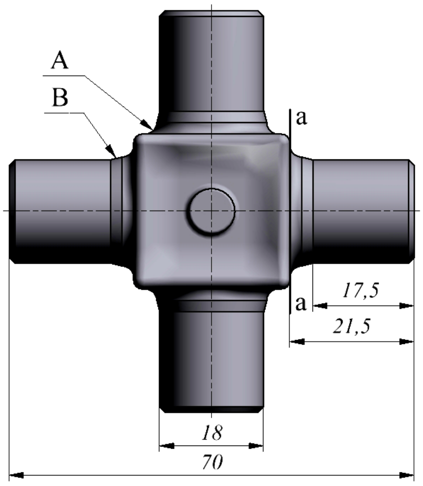

Fig.1. Dimensions and shape of cross shaft

The zone of shape transition from central part to the branches is indicated with A at Fig.1, while the zone of shape transition to cylindrical part for needle bearing is indicated with B.

The stress due to bending at the basic of branches of the cross (cross section a-a) is determined as

$$\sigma_s = \frac{32 \cdot T_{u2\max} \cdot \left(h_1 - \frac{h_2}{2} \right)}{\left(L - h_2 \right) \cdot \pi \cdot \left(d^3 - d_1^3 \right)} = 54.1 \text{ MPa}, \tag{1}$$

where $T_{u2\max}$ = 126.088 Nm is maximal value of torsion torque [7]. The shear stress at critical zone of cross shaft is determined as

$$\tau = \frac{4 \cdot F}{\pi \cdot \left(d^2 - d_1^2 \right)} = 9.93 \text{ MPa}, \tag{2}$$

where F =2.4 kN is force that act at branch of cross shaft [7]. Resulting stress is determined as

$$\sigma_e = \sqrt{\sigma_s^2 + 3 \cdot \tau^2} = 56.77 \text{ MPa}. \tag{3}$$

The determined value of stress presents the maximal value of stress at critical cross section. This value is not determined for the real shape of cross shaft that enclose the presence of stress concentrators, so for determination of stresses at real model of cross shaft the numerical method must be used. From the aspect that cross shafts can be made of different steel grades, some data obtained in exploitation indicate that it is beneficial if bending stress do not exceed 150 MPa [8].

Maximal stress act at the zone of cross shaft' section changing from central part to the branches and represent critical stress for the whole power transmitter. By the variations of geometrical parameters and repetitions of numerical calculations of stresses with invariant load conditions, the optimal fillet in critical section of cross shaft was created in order to obtain minimal stress concentration.

2.2. Stress state analysis by FEM method

The major of numerical methods for calculation at mechanical constructions are based on finite element method. The finite element method is used to precise analysis of real stress state at critical zones. The analysis by FEM method is much more precise in relation to analytic method. The finite element method provides possibility of fast repeated calculations after modifications of some design details of considered element. In this paper the simulation of load at cross shaft done using the software pack Autodesk Inventor Professional 2011 is presented. The analysis by finite element method require following procedure [9]: creation of geometric model, definition of material, discretization by finite elements, definition of support location and load limitations, the specification of location and characteristic of load, numeric calculation and interpretation of results.

The basic considered model for analysis by FEM method in this paper is created upon the real cross shaft. Geometric model made by Computer Added Design software packet is formed from simple geometrical shapes called geometric forms. The geometric model defines the real geometry of the considered element. The material of the all considered models in this paper is steel with following characteristic: elastic modulus E=$2 \cdot 10^5$ MPa, Poisson's ratio ν=0.287 and maximal stress limit is σ_{max}=275.8 MPa.

The three dimensional tetrahedral discretization with density variation is done at first stage of numerical model generation. The zone of shape transition as zone of interest is discretized by the finite elements with smallest dimensions (Fig. 2.).

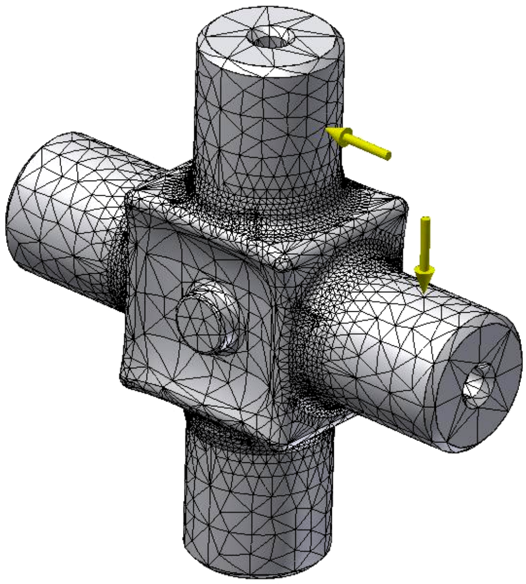

Fig.2. Discretization of analytical model

The border conditions are defined in according to theoretical considerations of stress state at cross shaft. The cross shaft is element with symmetry and four branches. The axes of the branches are in the same plain, forming the angle of 90^{o} by them. The every branch is loaded by the same force transmitted from the yoke by the bearings. In the reference [10] and [11] numeric analysis is done for quarter of the cross shaft, only one branch loaded by one of the forces, but the numeric analysis in this paper is done for the whole cross shaft. The central zone is fixed and the each branch is loaded by the equal force of F=2.4 kN.

The numeric calculation is done on the real model, and after that, modifications of shape transition zone is done for the aim of reduction of maximal stresses. The numeric calculations ware done on the modificated models and the obtained values of maximal stresses are compared in order to identify the design solution with minimal value of maximal stresses. On the bases of the evaluation of obtained results the modification of design solution of cross shaft with differences in shape and dimensions are done. Three different cases are considered:

- Case I – the zone of shape transition between the central part and branches is fillet with variation of radius and the fillet of 0.5 mm is done to the cylindrical part of branches for needle beadings.
- Case II – the zone of shape transition between the central part and branches is fillet with variation of radius and the chamfer of 0.5/45^{o} is done to the cylindrical part of branches for needle beadings.
- Case III – the zone of shape transition between the central part and branches is fillet with variation of radius and zone of shape transition to the cylindrical part of branches for needle beadings is 0.5x2 mm.

Every modification of cross shaft is analyzed under the same conditions and loads. The input parameters to numerical modelling software defined the static system, geometrical characteristics and load mode and border conditions. After the defining geometric model and input parameters the static structural analysis by numeric method is done.

2.3. Results of numeric analysis

The design solutions of cross shaft is done by variation of fillet at zone of shape transition from central part to the branches in diapason 0.5-2 mm with increase step of 0.25 mm. Visualisation of results of calculation of stresses for different modification of design with fillet of 0.5 mm is presented at Fig.3, Fig.5 and Fig.7.

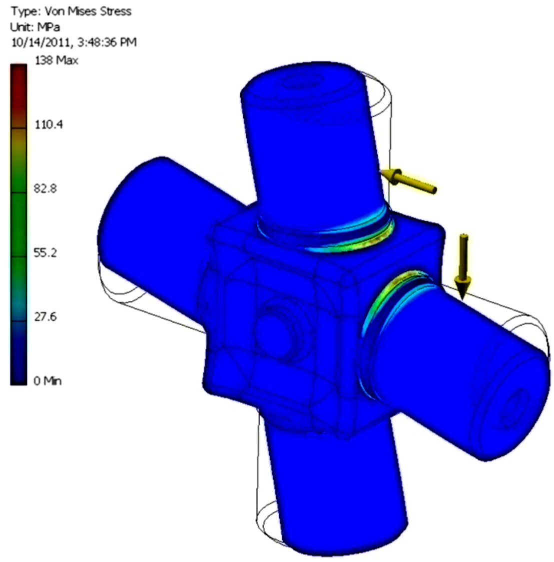

Fig.3. Finite element stress analysis of the cross shaft with fillet A of 0.5 mm and fillet B of 0.5 mm

Visualisation of results of calculation of stresses for different modification of design with fillet of 2 mm is presented at Fig.4, Fig.6 and Fig.8.

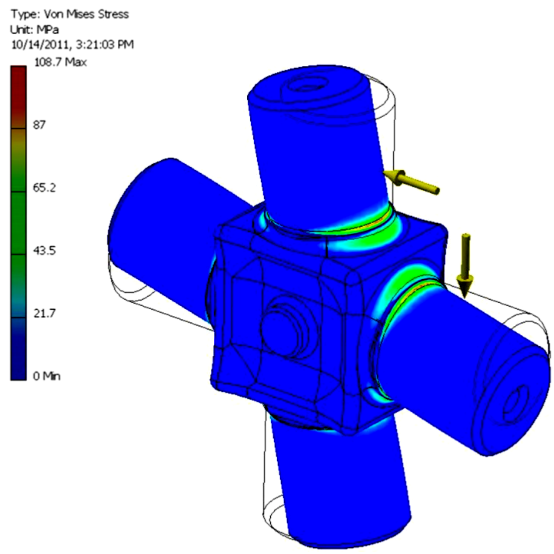

Fig.4. Finite element stress analysis of the cross shaft with fillet A of r=2 mm and fillet B of r=0.5 mm

Upon the presented visualisation of stresses at numeric model, especially for the considered zones of shape transitions at the basis of the branches of the cross shaft modifications of shape and fillet at those zones are done in order to reduce maximal stress level.

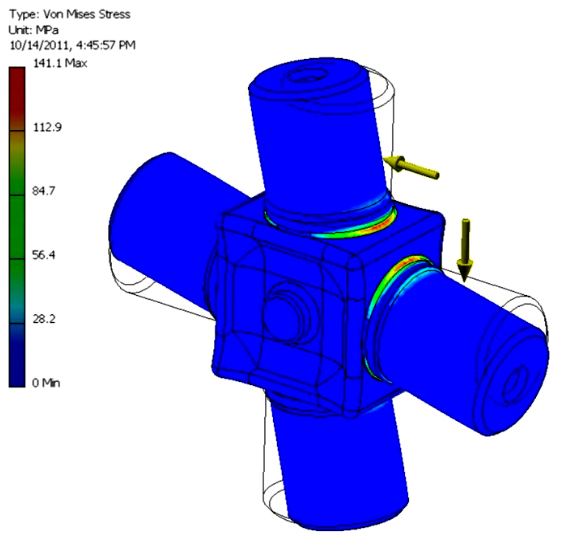

Fig.5. Finite element stress analysis of the cross shaft with fillet A of r=0.5 mm and chamfer B of 0.5/45º

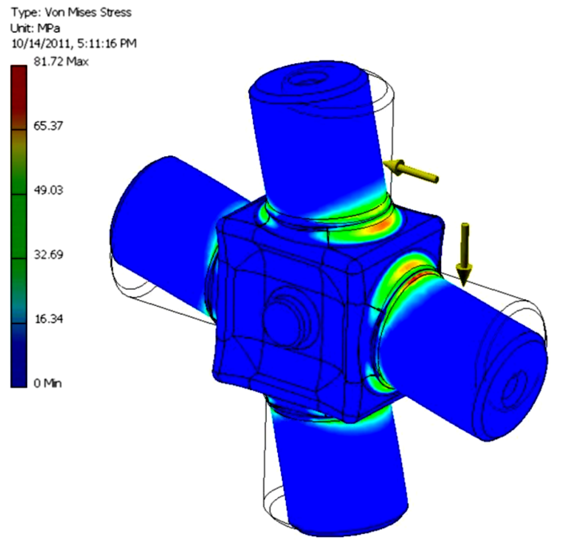

Fig.6. Finite element stress analysis of the cross shaft with fillet A of r=2 mm and chamfer B of 0.5/45º

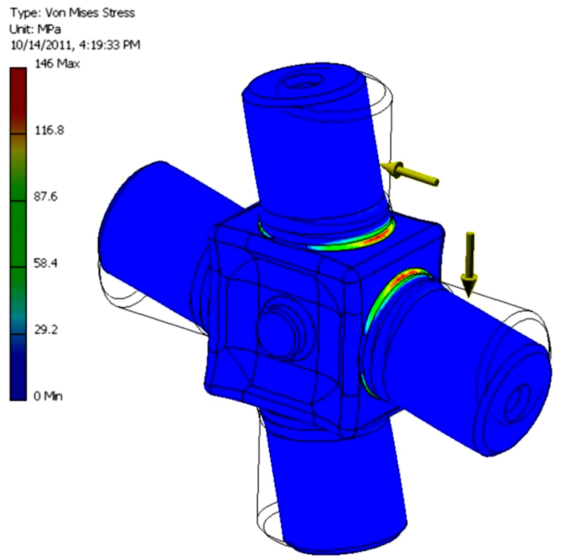

Fig.7. Finite element stress analysis of the cross shaft with fillet A of r=0.5mm and zone B of 0.5x2mm

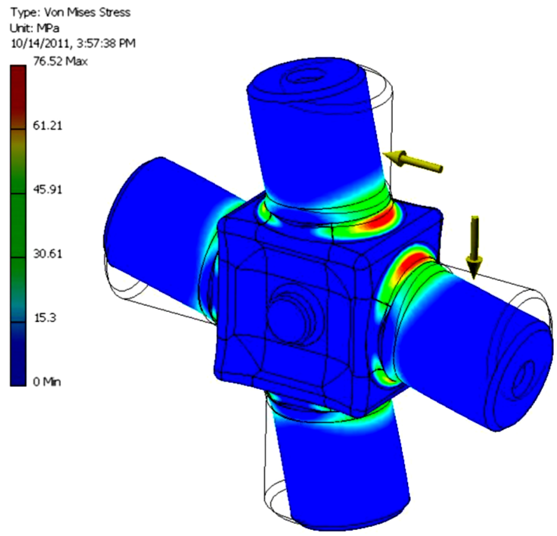

Fig.8. Finite element stress analysis of the cross shaft with fillet A of r=2mm and zone B of 0.5x2mm

The diagrams of stress variation as function of variation of fillet for three considered design solutions are presented at Fig.9, Fig.10 and Fig.11.

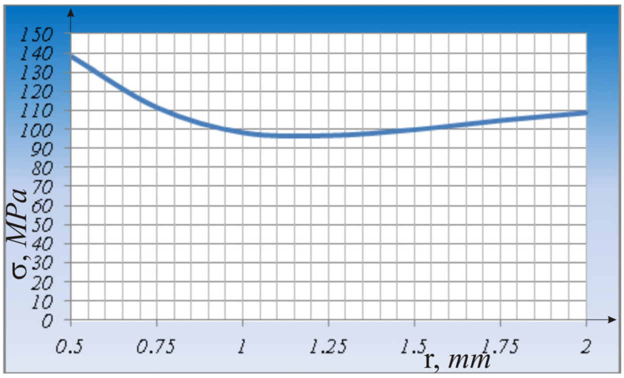

Fig.9. The diagram of maximal stress in function of fillet variation (Case I)

In the case I (Fig.9) with increase of fillet at the basis of branches to1.5 mm the variation of stress state is induced and stress concentration is decreased. With further increase of fillet, maximal stresses also increase and location of maximal stress is changing to zone where diameter decrease from bigger to smaller diameter of branches (Fig.4).

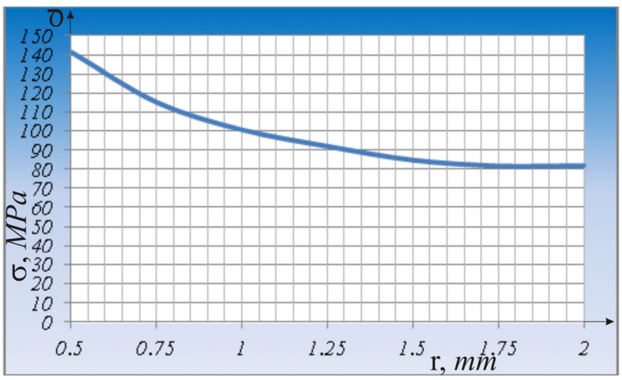

Fig.10. The diagram of maximal stress in function of fillet variation (Case II)

In the case II (Fig.10) with increase of fillet at the basis of branches to1.75 mm induced reduction of maximal stress. By the further increase of fillet, decrease of maximal stress is not obtained. Only localization of maximal stress is changed by this further increase of fillet (Fig.6).
In the case III (Fig.11) with increase of fillet at the basis of branches to 2.0 mm the monotony decrease of maximal stress level. The location of maximal stress is at zone of shape transition from central part to the branches.

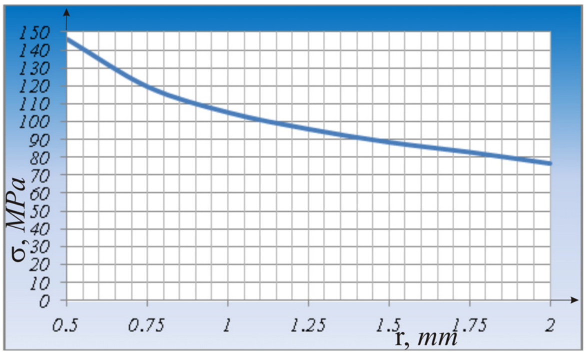

Fig.11. The diagram of maximal stress in function of fillet variation (Case III)

On the basis of the evaluation of obtained results from analysis of stress state at cross shaft of Cardan joint the optimal design solution with lowest maximal stress level can be identify. The values of maximal stresses at cross shaft for different design solutions with different combinations of shapes and dimensions of zone of shape transitions are presented at Tab.1. The design of cross shaft presented at Fig. 8 for the optimal design solution with lowest maximal stress of 76.52 MPa, is chosen.

Tab.1. The influence of geometric modifications to variation of stresses

Radii [mm]	0.5	0.75	1	1.25	1.5	1.75	2
Case I							
Stress [MPa]	138	111.4	98.13	96.57	99.73	104.7	108.7
Case II							
Stress [MPa]	141.1	115	100.5	91.79	84.63	81.54	81.72
Case III							
Stress [MPa]	146	119.5	105.2	95.83	88.46	82.92	76.52

3. CONCLUSION

The design and process of project development of power transmitter with Cardan joint must be done with great care due to the set of constructional requirements that must be fulfil by design solution. The results obtained by analytic calculations can not take as relevant in all cases because those calculations are done on simplified model. The method of numeric calculation by finite element analysis is the one of the methods that provides calculations on the mathematical models with real geometry [12]. For presented reason, the subject of research presented in this paper is forming of mathematical model for FEM analysis of stress variations at the zones of shape transitions from central part to branches, with geometric discontinuities, upon static load condition. The research presented in the paper implicate that maximal stresses at the basis of the branches of the cross shaft can be significantly reduced by modifications of shape and fillet at the zones of shape transitions. The increase of fillet, to some limit, induced reduction of stress concentration level. The further increase of fillet over the limit induced increase of maximal stresses. The conducted research implicate that optimal design solution of shape transition zone from the central part to the branches is one with bigger level of fillet and chamfer with angle of 45° to the cylindrical part for the base of needle bearing.
The considerations presented in this paper implicate that contemporary approach to analysis of load capacity of cross shafts provide relevant data, so as it put new perspectives in research of increase the load capacity of cross shaft by design modification.

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