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OPTIMIZATION OF CARDAN JOINT DESIGN FROM LOAD CAPASITY ASPECT

ABSTRACT: To the aims of providing safe and reliable mechanical elements and systems at reasonable prices, it is necessary to meet the basic requirements of construction related to its design that means, form, function, material and process development. Design of mechanical elements and systems resulted from the harmonization of those requirements and the task of designer is to identify the optimal design solution through this process. The optimization of Cardan joint design in order to reduce maximal stress level at its elements is presented in this paper. Numerical simulation of stress-strain state was done by software that is based on application of finite element method at model of Cardan joint. Different design of considered Cardan joints are analysed, and solution with minimal level of maximal stresses at critical zones of elements are identified. The optimization of Cardan joint is done as step-by-step process at each of its elements. The results of numerical simulation of stress-strain state at critical zones of Cardan joint elements implicate that small modification of design solution can resulted in significant reduction of maximal stress levels.

KEYWORDS: Cardan joint, design, stress-strain state, numerical simulation

INTRODUCTION

The basic function of Cardan shafts in power transmitters is to transmit power and torque between the elements of power group which are on certain distance, misaligned and that are not rigidly connected. They can be used between elements without coincide in axes or with variations of relative positions. The Cardan joint get its name after Italian mathematician Girolami Cardano, who invented this type of power transmitter. Cardan joints also are referred as universal or Hooke joints have been used for many years at mechanical systems. The Cardan shafts are used as part of power transmitter at machine systems and vehicles in order to provide stabile and reliable transmission of power and torque with flexibility in relative position of input and output shaft to the angle of 30° between axes. The Cardan shaft with two joints is usually used. Cardan joint can be used under high speed and heavy load exploitation conditions with high operating angle between axes. From the application aspect, Cardan shafts are essential at power transmission of cars and trucks, building and agricultural machinery. Cardan joints are

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used at: transport and agricultural machinery, cars and locomotives, radio-electronic devices, machine-tools, drills and pumps of oil industry, control mechanisms of aircrafts and helicopters, wood industry, textile industry and so on. The Cardan shafts are, also, used to deliver power and torque to the additional systems of machines, agricultural tractors for example, when flexibility of position is essential.

Simultaneous with development of agricultural and transport mechanical systems that uses Cardan shafts, rapid development of Cardan mechanisms and their increasing use is implicated. For vehicles and agricultural machines, which are subjected to significant change of relative position of some of its shafts during motion, the optimization of Cardan joint design is implicated to provide mechanism that does not react to changes of position of the shaft axis, and there by maintain safety and reliability and good exploitation properties of machine [1].

Cardan shafts are systems made up of elements, arranged and linked together. A system is characterized in that it can be delineated from its surroundings, with the links to the surroundings at the boundary of the system. A variety of factors affects the selection of the sub-systems. The elements of the Cardan shaft systems are shown at Fig. 1.

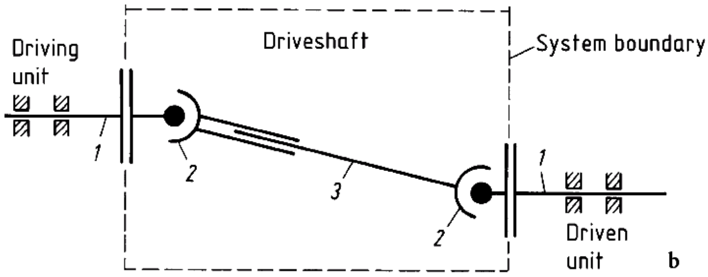


Figure 1 Scheme of Cardan shaft elements: 1 - connecting shaft, 2 - joints and 3 - intermediate shaft

Correct design of Cardan shaft and application, in terms of structural composition, provides greater freedom to designer during solving related requirements of development of power transmitters with Cardan joints. But, use of Cardan joint, in some cases, leads to high dynamic loads (at hydraulic power transmissions and so on).

The causes of failures and design of power transmitters with Cardan shafts are analysed by many researches. Hummel and Chassapis [2] researched on the design of the universal joints. They give some suggestions on the configuration design and optimization of universal joints with manufacturing tolerances [3]. Bayrakceken et al. [4] performed the fracture analysis of a universal joint yoke and a drive shaft of an automobile power transmission system. Spectroscopic analyses, metallographic analyses and hardness measurements are carried out for each part. For the determination of stress conditions at the failed section, stress analyses are also carried out by the finite element method (FEM). The reference considered modification of design of Cardan shaft in order to avoid failures during exploitation period.

By the analysis of information and data obtained in many studies related to this area referred the fact that exploitation reliability of Cardan shaft is directly determined, by reliability of needle bearing and cross shafts. Roller bearings and cross shafts work under very hard exploitative conditions due primary to high impacts loads. In the cause when Cardan shafts function with high operating angles, increase of inertial forces are induced that act on roller bearings and Cardan shaft altogether with external load. This leads to severe damages of roller bearings and cross shafts.

In most cases, the major influential factor to dimensions and reliability of Cardan shaft are, also, properties of its fork. In the study presented in this paper, the stress state at the joint fork under nominal load is analysed by numerical method. Finite element method is used to analyse the stress distribution in critical zone. On the bases of the obtained results the optimization of joint fork design was done.

Cross shaft is one of the most important parts of the Cardan joint. In significant number of cases, the size and lifetime of Cardan shaft highly depend on its cross shaft. Cross shaft is loaded to bending and torsion. It is proven that small changes of design of cross shaft can lead to significant reduction of critical stress. By the means of iterative design modification and repetition of the numerical simulation, favourable ration between design of the cross shaft and maximal levels of stress has been obtained.

Cardan joint fork, in dependence of type of connection with adjoining shaft can be equipped with bearing or flange. The flange couple the fork of the Cardan joint with second shaft. The coupling can be obtained also by groove joint or by welding. There are different designs of the Cardan joint fork with different types of joining to adjoining shaft. In many cases, the major influential factor to dimensions and reliability of Cardan shaft are the properties of its fork. In the study presented in this paper, the stress limit at the fork is calculated by numerical method. Finite elements method is used to analyse the stress distribution in critical zone. On the bases of the obtained results the optimization of fork design was done.

To the aim of better understanding of complex process of Cardan joint shaft design, this process is presented in the paper through evaluation of each element design separately. Functional characteristics of specific Cardan joint design are essential. As, those characteristics are primarily based on kinematic principals of Cardan mechanism the basis of Cardan joint kinematics will be briefly presented.

KINEMATIC OF CARDAN MECHANISMS

A universal joint is a positive, mechanical connection between rotating shafts, which are usually not parallel, but intersecting. They are used to transmit motion, power or both (Fig. 2)

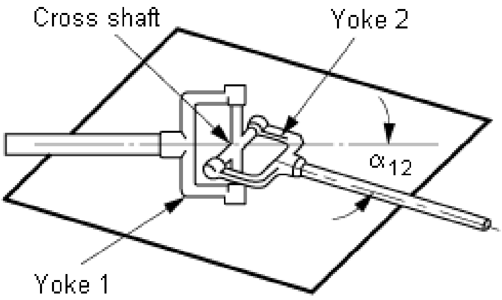


Figure 2 Scheme of single Cardan joint kinematic element

The simplest and most common type is called Cardan joint or Hooke joint. It is shown at Fig. 2 and it consists of two forks, one on each shaft, connected by a cross-shaped intermediate member called the cross shaft. The angle between the two shafts is called the operating angle and it is, in general, but not necessary, constant during operation. Good design practice requires low operation angles, often less than 25°, depending on the application. Independent of this guideline, mechanical interference in the construction of Cardan joints limits the operating angle to a maximum value that depends on its geometrical proportions. The main property of the Cardan mechanisms is possibility of changing the rotation speed ratio. Amplitude of periodical variation of rotation speed ratio depends on value of the angle between input and output shafts [5 and 6]. The relation between the rotation angles of input and output shafts is function of their' relative positions in the area. Independently of types and constructional solutions of Cardan mechanisms the basic kinematic relations are equivalent. The Cardan mechanism with angle α_{12} between shafts is presented at Fig. 2. If the rotation angle of input shaft is φ_1 then rotation angle of the output shaft is φ_2. The relation between those angles presents the basic kinematic principle that is given in the following form [5 and 6]:

$$\varphi_2 = \text{arctg}\left(\frac{\text{tg}\varphi_1}{\cos\alpha_{12}}\right). \tag{1}$$

The difference in value of angles φ_1 and φ_2 implicate the difference in rotational speeds of the corresponding shafts ($\omega_1 = \frac{\mathrm{d}\varphi_1}{\mathrm{dt}}$, $\omega_2 = \frac{\mathrm{d}\varphi_2}{\mathrm{dt}}$) and the value of that difference in rotational speeds can be obtained by differentiation of the equitation (1). By applying certain trigonometrical transformation the relation for rotation speed ratio of Cardan mechanism can be obtained [5 and 6]:

$$i_{21} = \frac{\omega_2}{\omega_1} = \frac{\cos\alpha_{12}}{1-\sin^2\alpha_{12}\cos^2\varphi_1}. \tag{2}$$

DESIGN OF CARDAN JOINT YOKE

Analytical and numerical calculation of stresses at model of real Cardan joint yoke (Fig. 3) was done at the beginning of this stage of research. After that, parameters of analysed model are modified and stress distribution was calculated by numerical method in order to identify the best design solution with the lowest level of stresses [7]. On the basis of the nominal input data and the presented load configuration, the calculation of maximal torque, so as the calculation of maximal load force at Cardan joint is done [7].

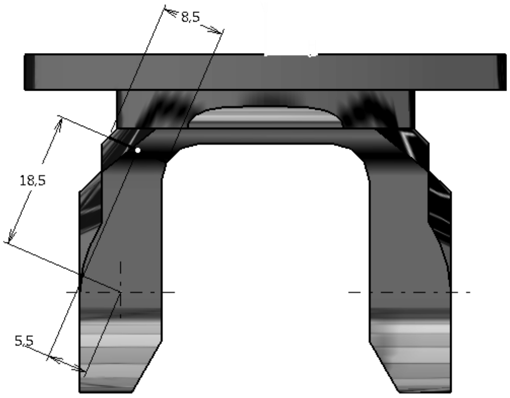


Figure 3 Cardan joint yoke

After this, the structural analysis by finite element method, by CATIA® software environment is done. The results calculated by numerical and analytical method were compared. Upon the relevant results of this analysis, the effect of design modifications to maximum stress was analysed. The stress distributions for six different designs of input yoke of Cardan shaft were analysed, by only two characteristic cases will be presented in this paper. The 3D tetrahedron discretization with variation in size of elements is used. The transition zone between yoke and cylindrical part was discretized by finest elements, as critical zone identified upon theoretical analysis. The boundary conditions were defined according to theoretical consideration of the input yoke of Cardan joint. The limits of movement were set on the surface which is overlapped on shaft. The load was set to force of 2509 N on the branch of the yoke for every design solution. The numerical model of Cardan joint yoke was completely defined and statically determined. Geometrical model of Cardan joint yoke was simplified, in this case, without profiling the branch of joint yoke (Fig. 4) and visualization of stress distribution is presented at Fig. 5 [7]

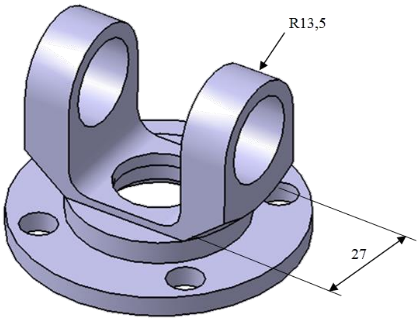


Figure 4 Simplified model joint yoke

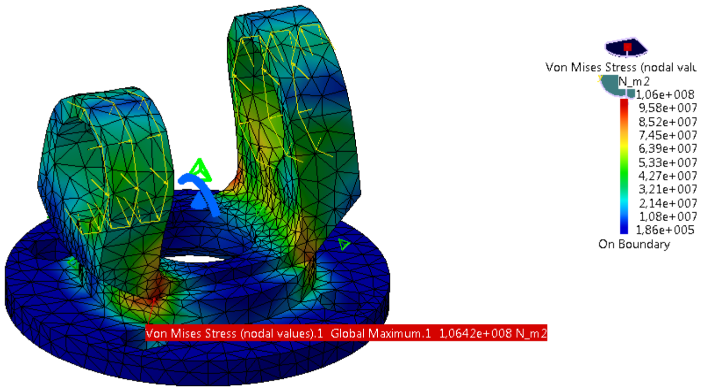


Figure 5 Visualization of Von Misses stress distribution at Cardan joint yoke

At the transition zones between the base of yoke and its cylindrical part, the stress concentration is caused and the maximum stress reaches the highest value. The difference between maximal stress obtained by analytical and numerical calculation is around 10%.

The design solution that provides lowest level of maximal stresses was identified. This design solution have profiled branches of the yoke and to fillet of 3 mm each at transition zone while the rest of the shape and dimensions remain the same as at other considered designs (Fig. 6). The visualization of Von Misses stress distribution is presented at Fig. 7.

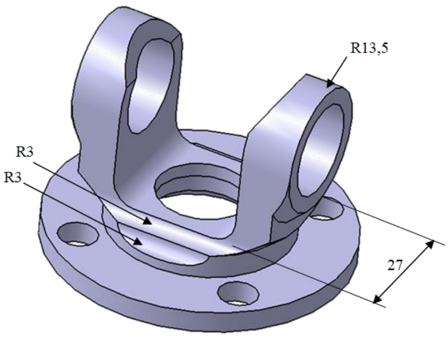


Figure 6 Design of Cardan joint yoke that provide lowest level of maximal stress

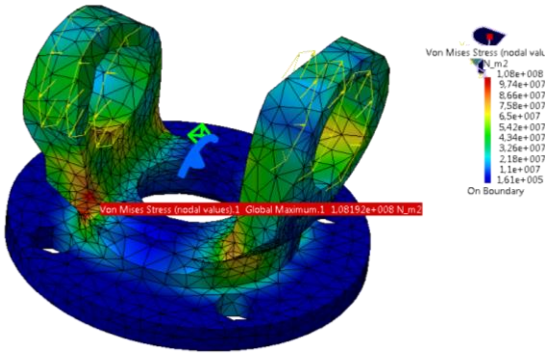


Figure 7 Visualization of Von Misses stress distribution at model with optimal design

Optimal design solution of Cardan joint yoke provides lowest level of maximal stresses. The position of zone with highest level of stress concentration was changed and transfer from the transition zone between yoke branches and cylindrical part to inner edge side of the base of branch yoke [7].

OPTIMAL DESIGN OF CARDAN JOINT CROSS SHAFT

Cross shaft is one of the most important, but also most sensitive parts of the Cardan joint. At this part of the paper procedure to determine optimal Cardan joint cross shaft design is presented. The real design solution of Cardan joint cross shaft is presented at Fig. 8 [8, 9 and 10].

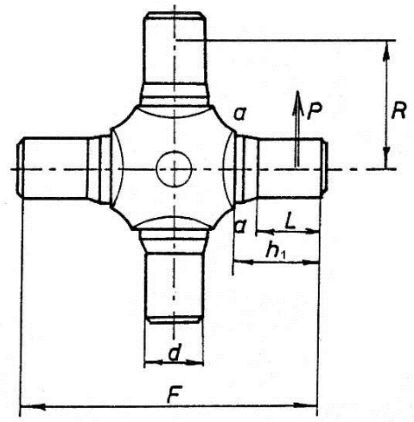


Figure 8 Design solution Cardan joint cross shaft

Cross shaft is loaded to bending and to torsion by torque. Bending stresses at the base of the cross shaft sleeve (critical section a-a, Fig. 8). At first, simplified model with missing of lubrication opening from real model was analysed. Due to the symmetry, only one quarter of cross shaft was considered. Design solution with basic geometrical dimensions of considered model is presented at Fig. 9. Distribution of Von Misses stress at considered one quarter of considered cross shaft is presented at Fig. 10 [8, 9 and 10].

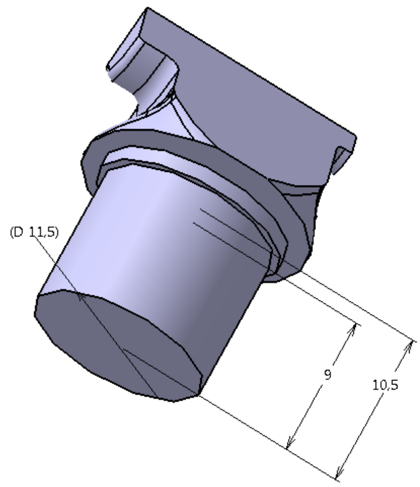


Figure 9 Part of the model of simplified cross shaft design solution

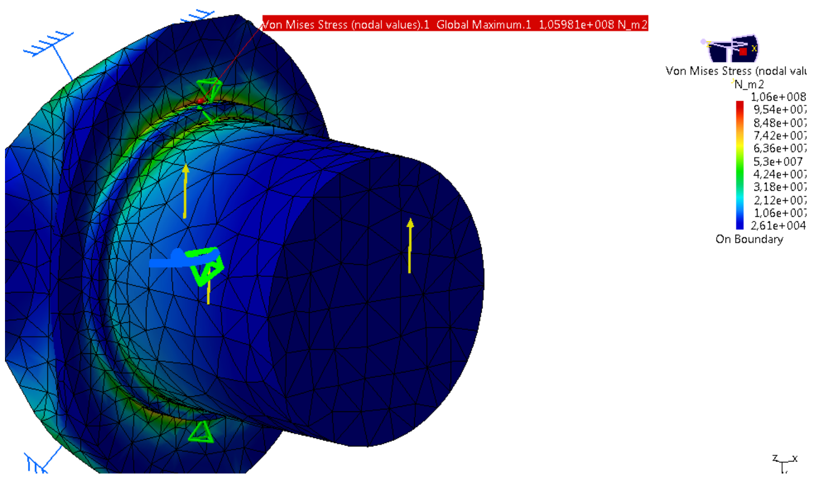
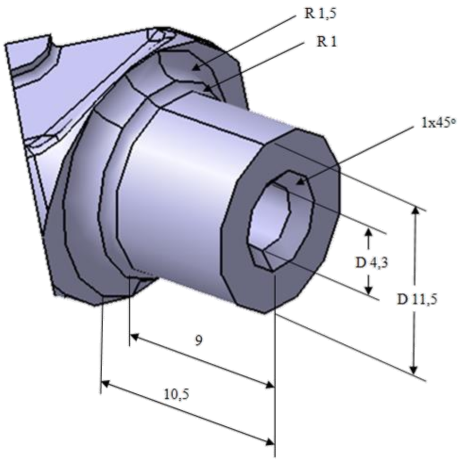


Figure 10 Visualization of Von Misses stress distribution at model with simplified cross shaft design solution

The maximum stress occurs at the root of the branches of the cross shaft and difference from the equivalent stress obtained by the analytical method is about 3%. As difference of values determined by numerical and analytical method is low, numerical model can be assumed as relevant and can be used for analysis of design modifications. The model of design solution with lowest level of maximal stresses is presented at Fig. 11, while distribution of Von Misses stress at this model is presented at Fig. 12.



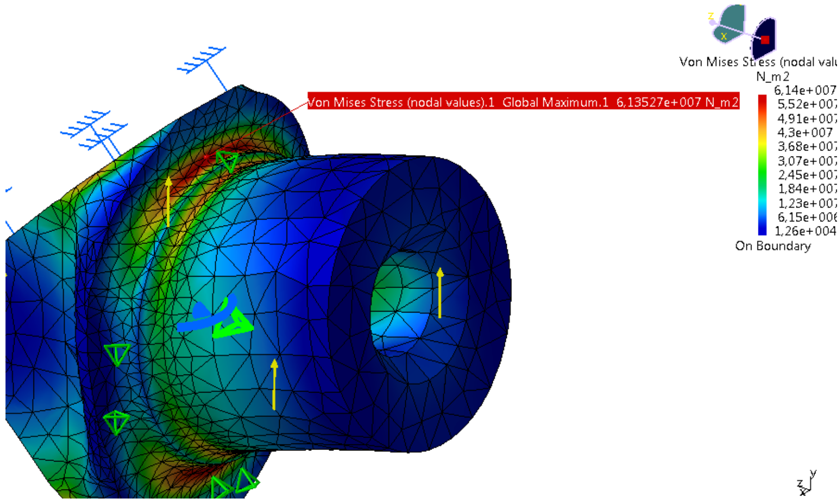


Figure 11 Part of the model of optimal cross shaft design solution

Figure 12 Visualization of Von Misses stress distribution at model with optimal cross shaft design solution

By increase of radius at root of cross shaft branches from 1 to 1.5 mm, change of stress distribution is caused. By this way, stress concentration at this zone of the considered model is also reduced [8, 9 and 10].

DESIGN MODIFICATION OF CARDAN SHAFT DRIVEN FORK

Calculation of stresses at Cardan joint fork is done on the basis of maximal torque and maximal force due to transmission of nominal power. Analytical and numerical calculation of stresses at Cardan joint driven fork was done at its real design and dimensions. After that, the modification of design was done and the stress distribution was obtained by numerical method in order to identify the optimal design with the lowest level of stresses. Model of the considered fork is presented at Fig. 13 [11].

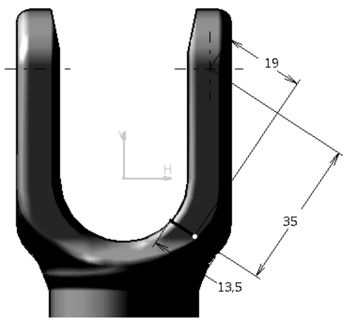


Figure 13 Model of the considered Cardan joint fork

Structural analysis by finite element method, made by CATIA® software is done and obtained results are put in correlations to values of stresses obtained analytically for the driven fork. The comparisons are done to evaluate the relevancies of the results. On the basis of the relevant results, the effect of design modifications to the maximum stress level was analysed. Fork of Cardan joint are under torque load at both sides with same intensity, so one half of the fork can be considered. The modifications of the design solution can be done at considered half due to the symmetry of element.

The stress distributions for four different design solutions of Cardan joint driven fork were analysed. Firstly, the three dimensional tetrahedron discretization with variation of dimensions of elements is used. The transition zone between fork and cylindrical part was discretized by finest elements while the rest of the analysed element was discretized by elements with higher dimension to reduce calculation time. The boundary conditions were defined according to theoretical considerations. The load was set to same force on the branch of the fork for every modification of design. The numerical model of Cardan driven fork was completely defined and statically determined. At first, model of design solution was simplified, without profiling the branches of driven fork (Fig. 14). The stress distribution at this model is presented at Fig. 15. Considered, simplified, model is different from the real design also by neglecting of fillet. Dimensions and geometrical characteristics of cross sections of Cardan joint fork branches are the same at whole its length [11].

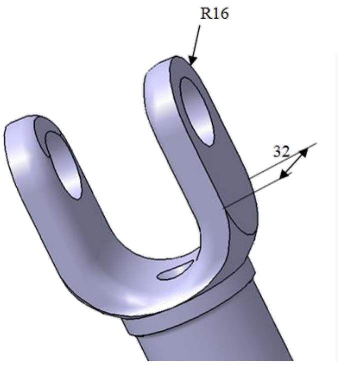


Figure 14 Simplified model of Cardan joint driven fork

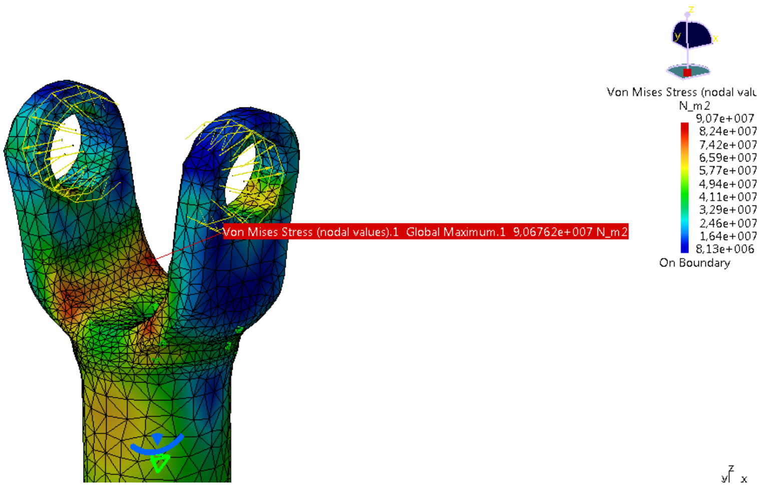


Figure 15 Visualization of Von Misses stress distribution at simplified model of Cardan joint driven fork

Maximal stress act at the base of the branches of Cardan joint fork at simplified model. The difference to the analytically obtained value is 4.5%. The optimal design solution is done by modification in that way to enlarge the cross section of the branches. By this way, dimensions of cross section rise at the end of the branches.

Model of optimal design solution of the considered Cardan joint driven fork is presented at Fig. 16. Visualisation of Von Misses stress distribution at model of optimal design solution is presented at Fig. 17 [11 and 12].

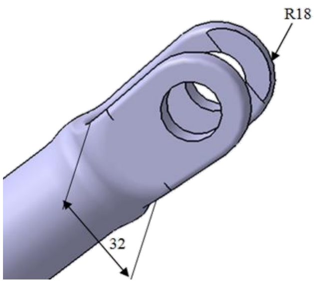


Figure 16 Optimal design of Cardan joint driven fork

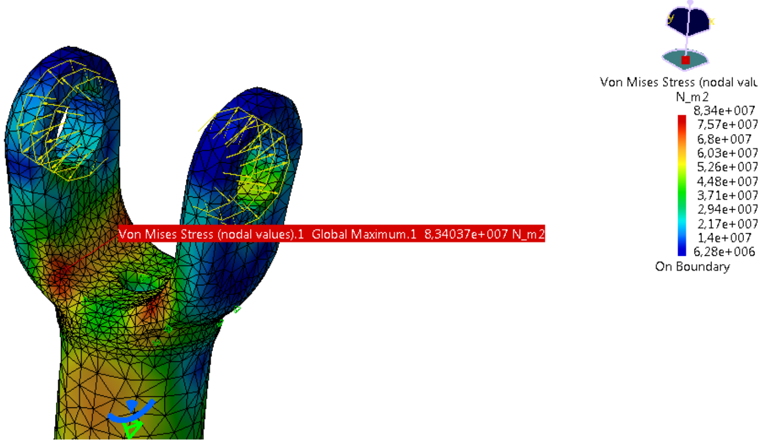


Figure 17 Visualization of Von Misses stress distribution at model of Cardan joint driven fork with optimal design

Obtained stress distribution at model with optimal design that is presented, showed that level of maximal stresses are reduced with 12% difference to analytically obtained values, so this modification of design is beneficial [11 and 12].

RESULTS AND DISCUSION

At the input yoke with increase of fillet, the higher level of stress can be reached. The variation in stress level can be up to the 30% at certain design solutions, even if the dimensions remained the same as for analytical calculation. Based on analysis of the Cardan joint cross shaft, it can be concluded that small design modifications can lead to significant changes of stress distribution at considered element.

By the examining the cross shaft, it was concluded that the grooves at zone of transition and base can reduce stresses up to 40%. It is implicated that the best option is using of two different radius fillets, with the smaller radius at the turn of a larger underlying sleeve. The evaluation of results must be done by using of other software and experimental testing.

The changing of the maximal stresses at considered driven fork of the Cardan joint is caused by changing of fillet at its ends, so as, by changing of the dimensions at cross section of its branches. The value of maximal stress can be reduced by 12% due to different design solutions. This modification of design and variation of geometrical parameters of the considered fork in relation to its effects on value of maximal stresses will be subject of future considerations.

The real model of Cardan joint shaft that was subjected to optimisation from load capacity aspect is presented at Fig. 18.

Figure 18 Model of real Cardan shaft that was subjected to optimization from load capacity aspect

As optimization of Cardan joint shaft design is done step-by-step at its specific elements, the optimal model is formed of elements with optimized designs. The optimisation is done from the load capacity aspect to form model of Cardan shaft that can replaced real Cardan shaft and provide reduction of stresses and by that increase its safety and reliability. The optimal model of Cardan joint shaft from the load capacity aspect is presented at Fig. 19.

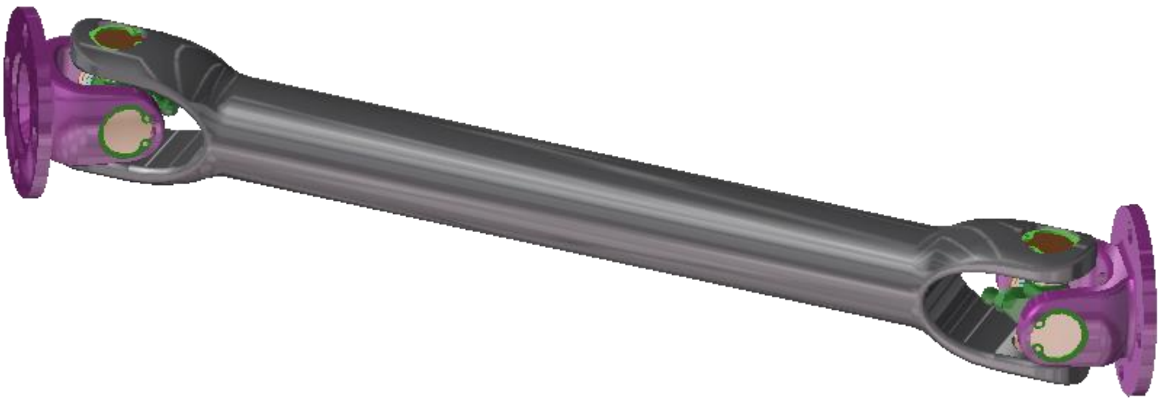


Figure 19 Model of Cardan shaft with optimal design from load capacity aspect

CONCLUSION

The optimization of design of Cardan shaft can be done from different aspects. In this paper the optimization from the aspect of load capacity is presented as one of the most important aspect that directly influent to its safety, reliability and working life. It is implicated that even small modification of Cardan shaft design can resulted in significant reduction of maximal stresses. As Cardan shafts are systems of arranged, functionally connected elements, optimization must be done by step-by-step modification of its every element [13].

Stress concentration can be reduced by fillets and by curvature of shape. The results obtained by analytical method cannot be reliably used in all cases as referent, because they are done at general, simplified model. By design solution with complex shape and keeping the same dimensions stress levels can be significantly reduced, that cannot be considered by analytical calculation [14].

The further complexity of shape with remain dimensions can induce the changing in stress values and it cannot be considered analytically. From that reason, the numerical calculation is favourable method because this method considered the influence of real design and its modifications. For the evaluation of the obtained results, it is recommended to use different software packages for the same models of design solutions and the same load configurations [15].

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