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LETTER

Non-iterative design of IIR multiple-notch filters with improved passband magnitude response

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Summary

A non-iterative method for the all-pass based design of Infinite Impulse Response (IIR) multiple-notch filters is proposed. This method ensures that specifications regarding positions of notch, left- and right-hand cutoff frequencies are exactly met, leading to a symmetric magnitude response about notch frequencies. Passband magnitude response can be improved by increasing the order of the corresponding all-pass filter.

KEYWORDS:
multiple-notch filter, passband magnitude response, all-pass filter, IIR filter

1 | INTRODUCTION

Digital multiple-notch filters, needed in a wide range of digital signal processing applications, can be designed either as IIR (Infinite Impulse Response) or FIR (Finite Impulse Response) filters. However, the order of the IIR multiple-notch filter is significantly lower compared to its FIR filter counterpart. Magnitude response of ideal multiple-notch filter can be expressed as

$$\left|H_{\text{ideal}}(e^{j\omega})\right| = \begin{cases} 0, & \omega \in \left\{\omega_{n,1}, \omega_{n,2}, \ldots, \omega_{n,K}\right\} \\ 1, & \text{otherwise} \end{cases}, \tag{1}$$

where $\omega_{n,i}$ denotes $i-th$ notch frequency position, while K is the number of notch frequencies. Since zero-bandwidth of an ideal multiple-notch filter cannot be realized, specifications of real filter's magnitude response also comprise 3 dB notch bandwidths BW_i defining left- and right-hand 3 dB cutoff frequencies $\omega_{l,i} = \omega_{n,i} - BW_i/2$, $\omega_{r,i} = \omega_{n,i} + BW_i/2$, for $i = 1, 2, \ldots, K$.

There are several approaches to the IIR multiple-notch filter design[1,2,3,4,5,6,7,8,9]. Realization of notch filter using cascaded second-order sections form structures inevitable leads to uncontrollable passband gains[2,6], while iterative design methods based on utilization of convex optimization[3,4,9] or genetic algorithms[5] exhibit high complexity especially for a greater number of notch frequencies. All-pass based IIR multiple-notch filter design methods first introduced in[1], transforms the specifications of multiple-notch filter magnitude response into those of the phase response of an corresponding all-pass filter.

Non-iterative all-pass based multiple-notch filter design methods, whose generalization is given in paper[2], leads to either a drift of notch frequencies from their specified positions or to asymmetric magnitude responses about notch frequencies. Latter leads either to prolonged transient response to sinusoidal interferences (final notch bandwidths are narrower then specified) or distortion of spectrum components close to notch frequencies (final bandwidths are wider then specified)[10]. Furthermore, none of non-iterative all-pass based multiple-notch filter design methods offers possibility of passband magnitude response improvement. This is due to the value of the order of the corresponding all-pass filter, which equals to double the number of notch frequencies. The magnitude frequency response of those filters equals to

$$\mathcal{M}(\omega) = \left|\cos\frac{\phi(\omega)}{2}\right|, \tag{2}$$

where $\phi(\omega)$ is the phase response of the all-pass filter. Having in mind that $\phi(\omega)$ is monotonically decreasing function in ω, magnitude response $\mathcal{M}(\omega)$ will exhibit monotonic behavior in passbands. Mentioned provides no possibility of further passband magnitude response improvement, nor can prescribed notch and cutoff frequencies positions (whose number equals to triple the number of notch frequencies) be exactly met.

In this paper, a new non-iterative all-pass based IIR multiple-notch filter design method, where the order of the all-pass filter is higher than the double the number of notch frequencies, is derived in an attempt to allow further improvement of passband magnitude response. Furthermore, specifications regarding positions of notch and cutoff frequencies are exactly met. To our knowledge, non-iterative approach to mentioned design problem does not appear to be considered in the existing literature.

The rest of the paper is structured as follows. In Sec. 2, specifications of IIR multiple-notch filter with symmetrical magnitude response about notch frequencies (attenuations at cutoff frequencies are the same and equal to 3 dB) are transformed to those of the phase response of the corresponding all-pass filter. The proposed non-iterative design method is considered in Sec. 3, while design examples are given in Sec. 4. Comparison with existing non-iterative all-pass based IIR multiple-notch filter design methods is given in Sec. 5. Finally, concluding remarks are given in Sec. 6.

2 | PROBLEM FORMULATION

Transfer function of the multiple-notch filter with K notch frequencies, whose design is considered in the paper, is assumed to be

$$H(z) = \frac{1}{2}\left[z^{-(L-2K)} + A_L(z)\right],\ L \geq 3K, \tag{3}$$

where $A_L(z)$ is a transfer function of $L-$th order stable all-pass filter

$$A_L(z) = z^{-L}\frac{P_L(z^{-1})}{P_L(z)},\ P_L(z) = 1 + \sum_{l=1}^{L} p_l z^{-l}. \tag{4}$$

The order of the all-pass filter $A_L(z)$ should be greater than or equal to triple the number of notch frequencies, i.e. $L \geq 3K$, to ensure that the prescribed notch and cutoff frequencies positions (whose number equals $3K$) are exactly met.

Denoting the phase response of the all-pass filter $A_L(z)$ with $\phi_A(\omega)$, after substitution $z = e^{j\omega}$ in Eq. (3), following is obtained:

$$H\left(e^{j\omega}\right) = e^{j\cdot 0.5[\phi_A(\omega)-(L-2K)\omega]}\cos\frac{\phi_A(\omega)+(L-2K)\omega}{2}. \tag{5}$$

Correspondingly, multiple-notch filter magnitude and phase responses are given by

$$\left|H\left(e^{j\omega}\right)\right| = \left|\cos\frac{\phi_A(\omega)+(L-2K)\omega}{2}\right|, \tag{6}$$

$$\phi_H(\omega) = \arg\left\{H\left(e^{j\omega}\right)\right\} = \frac{\phi_A(\omega)-(L-2K)\omega}{2} + \arg\left\{\cos\frac{\phi_A(\omega)+(L-2K)\omega}{2}\right\}. \tag{7}$$

Note, for $L = 2K$, Eq. (6) reduces to Eq. (2). Taking into account that $\phi_A(\omega)$ is monotonically decreasing function in $\omega \in [0,\ \pi]$, $\phi_A(0) = 0$ and $\phi_A(\pi) = -L\pi$, the magnitude response given by Eq. (6) will match the one given by Eq. (1) if

$$\phi_{A,\text{ideal}}(\omega) = \begin{cases} -(L-2K)\,\omega, & \omega \in \left[0,\ \omega_{n,1}\right) \\ -(L-2K)\,\omega - 2i\pi, & \omega \in \left(\omega_{n,i},\ \omega_{n,i+1}\right),\ i = 1, 2, \ldots, K-1 \\ -(L-2K)\,\omega - 2K\pi, & \omega \in \left(\omega_{n,K},\ \pi\right] \\ -(L-2K)\,\omega_{n,i} - (2i-1)\,\pi, & \omega = \omega_{n,i},\ i = 1, 2, \ldots, K \end{cases}. \tag{8}$$

From Eqs. (7) and (8), it concludes that phase response $\phi_{H,\text{ideal}}(\omega)$ of the ideal multiple-notch filter is linear in passbands. However, since zero-bandwidths of an ideal multiple-notch filter cannot be realized, specifications of the real multiple-notch filter's magnitude response, transformed to all-pass filters phase response specifications, can be formulated as

$$\phi_A(\omega) \approx -(L-2K)\,\omega - 2(i-1)\pi, \quad \omega \in \mathcal{P}_i \tag{9}$$

$$\phi_A(\omega_{n,i}) = -(L-2K)\,\omega_{n,i} - (2i-1)\,\pi, \quad i = 1, 2, \ldots, K, \tag{10}$$

where $\mathcal{P}_i$ denotes $i-$th passband frequencies

$$\mathcal{P}_i = \begin{cases} \{\omega \,|\, 0 \le \omega \le \omega_{l,1}\}, & i = 1 \\ \{\omega \,|\, \omega_{r,i-1} \le \omega \le \omega_{l,i}\}, & i = 2, 3, \dots, K \\ \{\omega \,|\, \omega_{r,K} \le \omega \le \pi\}, & i = K+1 \end{cases}.$$

Furthermore, if symmetric magnitude response about notch frequencies is of interest, following two equations should be included

$$\begin{aligned} \phi_A\left(\omega_{l,i}\right) &= \phi_A\left(\omega_{n,i}\right) + \pi/2, \\ \phi_A\left(\omega_{r,i}\right) &= \phi_A\left(\omega_{n,i}\right) - \pi/2, \end{aligned} \tag{11}$$

for $i = 1, 2, \dots, K$.

Eqs. (9), (10) and (11) are all constrains needed to be imposed on the all-pass filter's phase response, in order to obtain corresponding multiple-notch filter's magnitude response close to one in passbands and symmetric about notch frequencies.

Since the all-pass filter $A_L(z)$ is completely determined by the minimum phase transfer function $P_L(z)$, Eq. (4), and

$$\phi_A(\omega) = -L\omega - 2\phi_P(\omega), \tag{12}$$

where

$$\phi_p(\omega) = \arg\left\{P\left(e^{j\omega}\right)\right\} = -\arctan\frac{\sum_{l=1}^{L} p_l \sin l\omega}{1 + \sum_{l=1}^{L} p_l \cos l\omega}, \tag{13}$$

denotes the phase response of the transfer function $P_L(z)$, Eqs. (10) and (11) can be rewritten in terms of the phase response $\phi_P(\omega)$ as

$$\begin{aligned} \phi_P(\omega_{n,i}) + K\omega_{n,i} &= (2i-1)\,\pi/2, \\ \phi_P(\omega_{l,i}) + K\omega_{l,i} &= (i-1)\pi + \pi/4, \\ \phi_P(\omega_{r,i}) + K\omega_{r,i} &= i\pi - \pi/4, \end{aligned} \tag{14}$$

for $i = 1, 2, \dots, K$, while Eq. (9) becomes

$$\phi_P(\omega) + K\omega \approx (i-1)\pi, \;\; \omega \in \mathcal{P}_i, \tag{15}$$

for $i = 1, 2, \dots, K+1$.

Having in mind previous discussion, design problem of multiple-notch filter with magnitude response symmetrical about notch frequencies reduces to determination of the unknown coefficients p_l, $l = 1, 2, \dots, L$, such that Eqs. (14) and (15) are satisfied.

3 | THE DESIGN METHOD

A mathematical relationship between coefficients and the phase response of the minimum phase transfer function $P_L(z)$, derived from (13)

$$\sum_{l=1}^{L} p_l \sin\left(\phi_P(\omega) + l\omega\right) + \sin\phi_P(\omega) = 0, \tag{16}$$

allows formulation of constraints regarding positions of notch, left- and right-hand cutoff frequencies given by (14), as linear equality constraints

$$\mathbf{Q}\mathbf{p}_1 + \mathbf{S}\mathbf{p}_2 = \mathbf{b}, \tag{17}$$

on unknown coefficients vector $\mathbf{p} = \begin{bmatrix}\mathbf{p}_1 \\ \mathbf{p}_2\end{bmatrix} = \begin{bmatrix}\left[p_1, \, p_2, \, \dots, \, p_{3K}\right]^{\mathrm{T}} \\ \left[p_{3K+1}, \, p_{3K+2}, \, \dots, \, p_L\right]^{\mathrm{T}}\end{bmatrix}$, where $\mathbf{Q} = \left[q_{ik}\right]$ is the $3K \times 3K$ square matrix, $\mathbf{S} = \left[s_{ik}\right]$ is the $3K \times (L-3K)$ matrix, while $\mathbf{b} = \left[b_1, \, b_2, \, \dots, \, b_{3K}\right]^{\mathrm{T}}$ is $3K \times 1$ column vector, such that

$$b_i = \begin{cases} -\cos K\omega_{n,i}, & i \le K \\ -\sin\left(K\omega_{l,i-K} - \pi/4\right), & K < i \le 2K \\ -\sin\left(K\omega_{r,i-2K} + \pi/4\right), & i > 2K \end{cases}, \tag{18}$$

$$q_{ik} = \begin{cases} \cos\left(\omega_{n,i}(K-k)\right), & i \le K \\ \sin\left(\omega_{l,i-K}(K-k) - \pi/4\right), & K < i \le 2K \\ \sin\left(\omega_{r,i-2K}(K-k) + \pi/4\right), & i > 2K \end{cases} \tag{19}$$

and

$$s_{ik} = \begin{cases} \cos\left(\omega_{n,i}(2K+k)\right), & i \le K \\ -\sin\left(\omega_{l,i-K}(2K+k) + \pi/4\right), & K < i \le 2K \\ -\sin\left(\omega_{r,i-2K}(2K+k) - \pi/4\right), & i > 2K \end{cases}. \tag{20}$$

Considering Eq. (6), it is reasonable to adopt the following passband magnitude error function

$$E_p(\omega) = \sin\frac{\phi_A(\omega) + (L-2K)\omega}{2}, \tag{21}$$

which, employing Eq. (12), becomes

$$E_p(\omega) = -\sin\left(\phi_P(\omega) + K\omega\right). \tag{22}$$

Since $\phi_P(\omega) + K\omega = \arg\left\{e^{jK\omega} \cdot P_L\left(e^{j\omega}\right)\right\}$, employing trigonometrical identity $\sin(\arg\{\cdot\}) = \frac{\Im\{\cdot\}}{|\cdot|}$ and Eq. (17), Eq. (22) can be rewritten as

$$E_p(\omega) = -\frac{u(\omega) - \mathbf{r}(\omega)\mathbf{p}_2}{\left|P_L(e^{j\omega})\right|}. \tag{23}$$

where $\Im\{\cdot\}$ is the imaginary part function, and

$$\begin{aligned} u(\omega) &= \sin K\omega + \mathbf{c}_1(\omega)\mathbf{Q}^{-1}\mathbf{b}, \\ \mathbf{r}(\omega) &= \mathbf{c}_1(\omega)\mathbf{Q}^{-1}\mathbf{S} - \mathbf{c}_2(\omega), \end{aligned} \tag{24}$$

where

$$\begin{aligned} \mathbf{c}_1(\omega) &= \left[\sin(\omega(K-1)),\ \sin(\omega(K-2)),\ \dots,\ \sin(\omega(K-3K))\right], \\ \mathbf{c}_2(\omega) &= \left[\sin(\omega(K-(3K+1))),\ \sin(\omega(K-(3K+2))),\ \dots,\ \sin(\omega(K-L))\right]. \end{aligned} \tag{25}$$

Discrete unconstrained optimization problem defined by

$$\underset{\mathbf{p}_2}{\text{minimize}} \quad \sum_{m=1}^{M} W(\omega_m) E_p^2(\omega_m, \mathbf{p}_2), \quad \omega_m \in \mathcal{P} = \bigcup_{i=1}^{K+1} \mathcal{P}_i \tag{26}$$

where $M \gg (L-3K)$ denotes total number of equidistant frequencies ω_m in passbands, and $W(\omega)$ is the positive weighting function, can be suboptimally solved for $L > 3K$ using iterative Steiglitz-McBride scheme [11]. However, it shows that satisfactory results can be obtained by non-iterative approach if the optimization problem given by Eq. (26) is simplified by neglecting denominator of the passband magnitude error function, that is

$$\underset{\mathbf{p}_2}{\text{minimize}} \quad \sum_{m=1}^{M} W(\omega_m)\left[u(\omega_m) - \mathbf{r}(\omega_m)\mathbf{p}_2\right]^2. \tag{27}$$

Solving Eq. (27) for the unknown coefficient vector $\mathbf{p}_2$ gives

$$\mathbf{p}_2 = \left(\mathbf{R}^{\mathrm{T}}\mathbf{W}\mathbf{R}\right)^{-1}\mathbf{R}^{\mathrm{T}}\mathbf{W}\mathbf{u}, \tag{28}$$

where $\mathbf{W}$ is $M \times M$ diagonal matrix with elements $W(\omega_m)$ on its diagonal, $\mathbf{R}$ is $M \times (L-3K)$ matrix with rows $\mathbf{r}(\omega_m)$ and $\mathbf{u}$ is $M \times 1$ column vector with elements $u(\omega_m)$, for $m = 1, 2, \dots, M$. Once $\mathbf{p}_2$ is known, coefficients vector $\mathbf{p}_1$ can be determined using Eq. (17) as

$$\mathbf{p}_1 = \mathbf{Q}^{-1}\left(\mathbf{b} - \mathbf{S}\mathbf{p}_2\right). \tag{29}$$

4 | DESIGN EXAMPLES

The efficiency of the proposed design method will be verified by designing two multiple-notch filters of various orders and different passband-wise constant weighting functions $W(\omega)$. For all examples, passband magnitude responses and phase errors:

$$\Delta\phi_H(\omega) = \phi_H(\omega) - \phi_{H,\text{ideal}}(\omega) = \frac{\phi_A(\omega) - \phi_{A,\text{ideal}}(\omega)}{2}$$

of obtained multiple-notch filters are given. Taking into consideration Eqs. (7) and (8), it follows that the phase response of the IIR multiple-notch filter obtained by proposed method approximate $-2(i-1)\pi - \omega(L-2K)$, which is linear function of frequencies, in i-th passband, for $i = 1, 2, \ldots, K+1$. Since, attenuation at cutoff frequencies equals 3 dB, error by which obtained multiple-notch filter phase response differs from the ideal linear phase equals $\pm\pi/4$ at cutoff frequencies.

For $L = 3K$, prescribed notch, left- and right-hand cutoff frequency positions will be exactly met, however attenuation in some passband may be larger than 3 dB. This is usually a case when all notch frequencies (but small number of them) are located close to 0 or π rad, and width of passband between notch frequencies is relatively small.

Example 1: Design of IIR multiple-notch filter with specifications regarding notch frequencies positions and 3 dB notch bandwidths given by: $\omega_{n,1} = 0.15\pi$, $\omega_{n,2} = 0.275\pi$, $BW_1 = BW_2 = 0.05\pi$, for $L = 6, 7, 8$ and $W(\omega) = 1$.

Obtained passband magnitude responses and phase errors are given in Fig. 1a), and Fig. 1b). For given specifications regarding

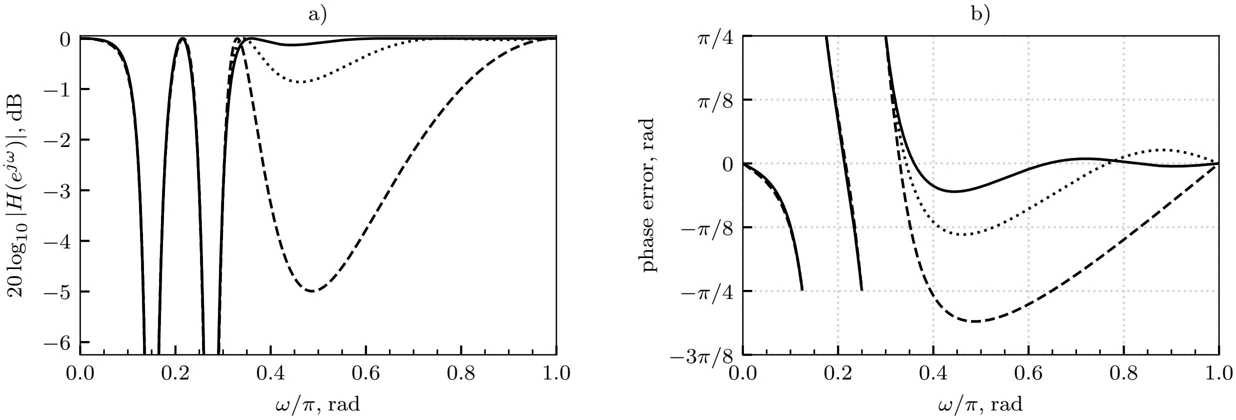

FIGURE 1 Example 1. a) Passband magnitude responses in dB, b) Passband phase errors in radians, for $L = 6$ (dashed line), $L = 7$ (dotted line) and $L = 8$ (solid line), obtained by proposed design method for $\omega_{n,1} = 0.15\pi$, $\omega_{n,2} = 0.275\pi$ and $BW_1 = BW_2 = 0.05\pi$.

notch frequencies positions and notch bandwidths, obtained transfer function for $L = 6$ is not suitable to be used as multiple-notch filter, which can be observed in Fig. 1a), due to high ripple value in the third passband. By increasing the order of the corresponding all-pass filter, passband phase error decreases, i.e. passband magnitude response improves. This improvement is followed by the decrease in maximum pole radius value, since it equals to 0.9124, 0.9104 and 0.9082, for $L = 6$, $L = 7$ and $L = 8$, respectively.

Example 2: Design of IIR multiple-notch filter with specifications regarding notch frequencies positions and 3 dB notch bandwidths given by: $\omega_{n,1} = 0.2\pi$, $\omega_{n,2} = 0.4\pi$, $\omega_{n,3} = 0.75\pi$, $BW_1 = BW_3 = 0.05\pi$, $BW_2 = 0.1\pi$, for $L = 9, 12, 18$ and

$$W(\omega) = \begin{cases} 2, & \omega \in \mathcal{P}_1 \\ 5, & \omega \in \mathcal{P}_2 \\ 5, & \omega \in \mathcal{P}_3 \\ 3, & \omega \in \mathcal{P}_4 \end{cases}$$

Obtained passband magnitude responses and phase errors are given in Fig. 2a), and Fig. 2b). As in the previous example, with increase of the all-pass filter order, the overall passband magnitude response improves. However, in this example, the maximum pole radius value increases and is equal to 0.9245, 0.9291 and 0.9464, for $L = 9$, $L = 12$ and $L = 18$, respectively.

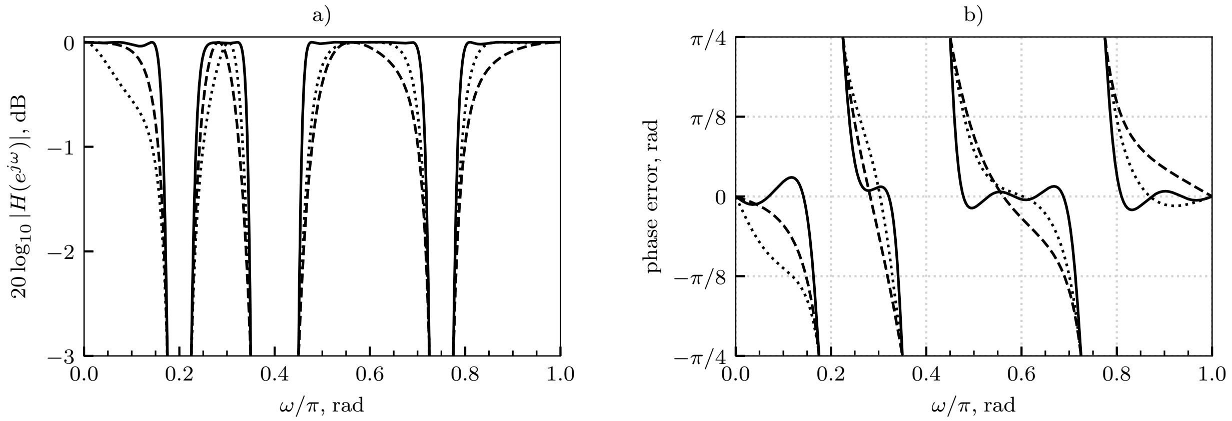

FIGURE 2 Example 2. a) Passband magnitude responses in dB, b) Passband phase errors in radians, for $L = 9$ (dashed line), $L = 12$ (dotted line) and $L = 18$ (solid line), obtained by proposed design method for $\omega_{n,1} = 0.2\pi$, $\omega_{n,2} = 0.4\pi$, $\omega_{n,3} = 0.75\pi$, $BW_1 = BW_3 = 0.05\pi$ and $BW_2 = 0.1\pi$.

5 | COMPARISON TO OTHER NON-ITERATIVE DESIGN METHODS

Proposed design method will be compared to two non-iterative methods guaranteeing specifications regarding notch frequencies positions are exactly met, namely *Method I* and *Method II* (apart from notch frequency positions, these methods guarantee that obtained left- and right-hand cutoff frequencies positions equal those specified, respectively) from[2]. The order of the all-pass filter utilized by those methods equals double the number of notch frequencies.

Magnitude responses of multiple-notch filter: $\omega_{n,1} = 0.2\pi$, $\omega_{n,2} = 0.6\pi$, $BW_1 = 0.1\pi$ and $BW_2 = 0.05\pi$, obtained by proposed procedure ($L = 10$, $W(\omega) = 1$), *Methods I* and *II*[2] are given in Fig. 3a). Corresponding passband phase errors are given in Fig. 3b).

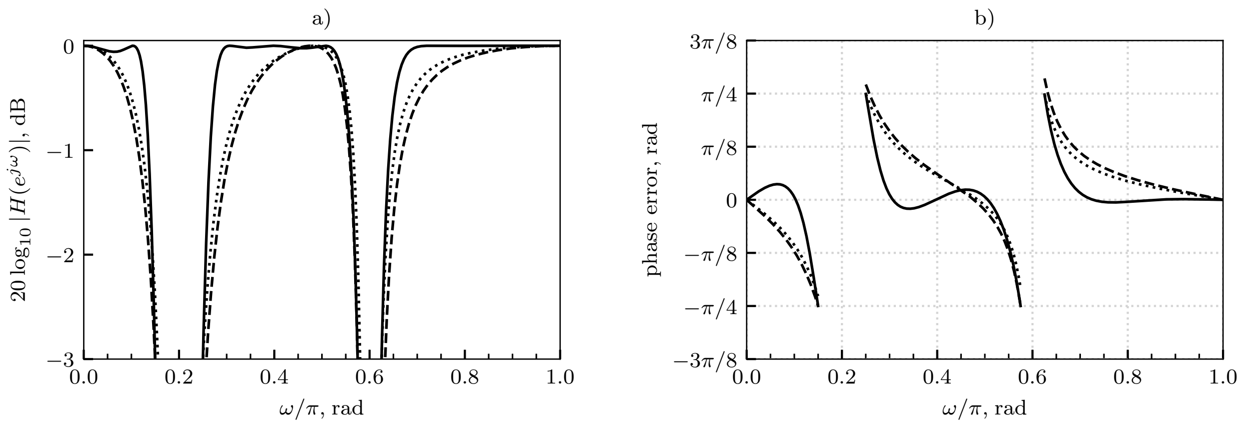

FIGURE 3 a) Passband magnitude responses in dB, b) Passband phase errors in radians, obtained by proposed (solid line, $L = 10$, $W(\omega) = 1$), *Methods I* and *II*[2] (dotted and dashed line, respectively) for $\omega_{n,1} = 0.2\pi$, $\omega_{n,2} = 0.6\pi$, $BW_1 = 0.1\pi$ and $BW_2 = 0.05\pi$.

It is clear that magnitude response of multiple-notch filter obtained by proposed method is better compared to those obtained by utilizing *Methods I* and *II*[2]. The maximum pole radiuses equal to 0.9039, 0.9136, and 0.9321, in the case of proposed, *Methods I* and *II*[2], respectively. Mentioned magnitude response improvement, of course, comes at the cost of somewhat higher all-pass filter order. Passband magnitude response as well as the passbands phase error, obtained by proposed design method, exhibit ripples, which is not the case when *Methods I* and *II*[2] are used.

6 | CONCLUSION

The main advantage of the proposed all-pass based IIR multiple-notch filter design method lies in its non-iterative nature. Furthermore, it provides the possibility to improve passband magnitude response by varying the order of the corresponding all-pass filter, which has to be greater than or equal to triple the number of notch frequencies in order to exactly meet specifications regarding positions of notch, left- and right-hand cutoff frequencies. In this way stability issues are avoided to be considered, i.e. there is no need to impose inequality constraints on unknown coefficients vector. Optimization problem derived by simplifying starting objective function has the closed-form solution to unknown coefficients vector, while at the same time satisfactory results are obtained.

Financial disclosure

None reported.

Conflict of interest

The authors declare no potential conflict of interests.

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