DESIGN OF A STATIONARY ASYMMETRIC SOLAR CONCENTRATOR FOR HEAT AND ELECTRICITY PRODUCTION

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ABSTRACT
A response of world community to temperature increase on the global level may be increased use of solar energy for heating, cooling, and electricity production. The paper presents a mathematical model for stationary asymmetric solar concentrators that would use solar energy to generate electricity and heat. The mathematical model would handle its optical, energy, and economical analyses. Then, the paper gives the simulation results for operation of these devices for Kragujevac, Serbia at 44 deg. north latitude for different aperture angle of these concentrators. The design with the fastest payback for this latitude and climate characteristics would be suggested to be constructed.

KEY WORDS
Modelling, simulation, artificial intelligence, and neural networks

1. Introduction

Temperature increase has been constantly recorded on the global level. The increase is due to green-house effect because of CO2 emission from different sources in atmosphere. Usually this emission is blamed to combustion of fossil fuels mainly for heating and electricity production. A response to such a situation of world community may be increased use of solar energy for heating, cooling, and electricity production. Solar energy is source of all life on the earth and it is in abundance but dispersed.

The subject of this research (in Centre of heating, air conditioning and solar energy of Mechanical Engineering Faculty at Kragujevac in Kragujevac University in Serbia) is design optimization of stationary asymmetric solar concentrator for heat and electricity production with geometric concentration of up to 7.

The stationary solar concentrators have advantages over tracking concentrators as they can be part of building façade and used in building and city architecture. In addition, these stationary solar concentrators have an advantage over ordinary solar hybrid collectors that expensive photovoltaics are replaced by cheap reflecting Al film.

This type of concentrators was already subject of intensive research [1,2]. The paper presents a mathematical model for these collectors that would handle analyses of its optical, energy, and economic characteristics [3]. Then, the paper gives the simulation results for operation of these devices for Kragujevac, Serbia at 44 deg. north latitude for different aperture angles. These results would reveal the best concentrator construction with respect to payback of investment taking into account the energy revenue.

2. Concentrator Design

2.1 Design description

A stationary asymmetric solar concentrator shown in Fig.1 consists of a reflector and absorber. The reflector has partially parabolic and partially cylindrical shape. The absorber has plate shape. This is basically aluminum covered with photovoltaic material. Inside the absorber, there are copper pipes where the heat transfer fluid flows.

Figure 1. Position of the concentrator. The concentrator is positioned to have the reflector trough in the direction east-west, while the reflector surface faces south (valid for the north hemisphere)

The concentrator operates as the following. The direct solar radiation falls on the different parts of the reflector surface. The reflector surface reflects the solar radiation toward the absorber. All reflected solar radiation hits and concentrates to the absorber. In the absorber, the concentrated solar radiation is transformed into electricity and heat. The electricity is directed to the battery. The
heat is collected by the heat transfer fluid and directed to the storage tank.

![Diagram](image)

**Figure 2. Vertical cut through the concentrator in the plane perpendicular to the direction east-west**

### 2.2 Concentrator position

The position of the concentrator is shown in Fig.1 and Fig.2. Figure 2 shows the vertical cut of the concentrator in Fig.1. The concentrator is positioned to have the reflector trough in the direction east-west, while the reflector surface faces south (valid for the north hemisphere). The vertical location angle $\beta_1$ of the concentrator facing south is shown in Fig. 3. The figure reveals that $\beta_1$ is the angle between absorber and horizontal direction due south. This angle is calculated as

$$\beta_1 = 66.25^\circ - \phi + \delta.$$  \hspace{1cm} (1)

Here, $\phi$ stands for the latitude and $\delta$ stands for the aperture angle of the concentrator. Figure 3 shows the vertical location angle of the concentrator vs. the aperture angle of the concentrator when the concentrators are located in Kragujevac, Serbia with $\phi = 44^\circ$.

![Diagram](image)

**Figure 3. Vertical location angle of the concentrator vs. the aperture angle of the concentrator**

### 2.3 Concentrator vs. conventional panels

The conventional solar panels only use half of their available surface area, the bottom half facing downward and in contact with opaque insulation. Because the heat-transfer fluid within a panel is easily able to accept more energy than is incident on the top surface of a conventional panel, there is illumination of both sides of the panel with non-imaging reflectors to allow a single absorber plate to be used instead of the two standard single-sided absorber plates. This represents a substantial cost and materials saving.

Such a design approach has a number of implications: (a) The panel must be substantially thicker because of the necessity for reflective optics to fit underneath; this can be an aesthetic issue. (b) The heat loss mechanism underneath the panel is different to that of the top surface. (c) Additional reflector material must be costed and the cost and pollution compared to the replaced panel.

![Diagram](image)

**Figure 4. Light ray path toward the concentrator (the cross cut of parabolic-cylindrical concentrator with one parabolic and cylindrical reflector)**

### 2.4 Light ray paths

Figure 4 shows the light ray paths toward the absorber of the concentrator. In this figure, we show the travel path of several light rays. Basically, light rays may hit the absorber from the front side, from the back side, and may miss the absorber. In Fig.4, the light rays are denoted as 1, 2, 3, Oa, and Ob. Only incident light ray 1 has direction that is parallel to the parabola focal plane (OF). Then, light ray 1 enters point T under aperture angle $\delta$. This is the angle between the focal plane and the tangent on the parabola in point T. This is also the angle between the tangent of the parabolic reflector and the line TF. The ray is reflected under the same angle $\delta$ from the parabolic surface to reach focal point F (that lays on the absorber). Incident ray 2 also hits the reflector of the concentrator at the rim point. Furthermore, the ray is reflected and then hits the absorber below focal point F. Incident light ray 3 hits the reflector of the concentrator at the rim point. Furthermore, the ray is reflected and then hits the cylindrical surface below focal point F, then, it is again reflected and hits the back side of the absorber. Incident light ray Oa hits the reflector of the concentrator. Furthermore, the ray is reflected and then it is discarded from the concentrator. Light ray Ob does not hit the reflector of the concentrator. Then, it travels outside of the concentrator without hitting it.
3. Mathematical Model

Our objective is to find the economically optimum design of the solar concentrator with parabolic and cylindrical reflector.

3.1 Geometry

Figure 5 gives detailed geometry of the concentrator. From geometry in the figure, angle \( \beta \) is equal to angle \( \gamma \), because “n” is perpendicular to tangent “t”, and “d” is perpendicular to the light ray. It is possible to have the following relationship

\[ 90 = \beta + (90 - \gamma) + \beta. \] (2)

Here, \( \gamma \) stands for the angle between the focal plane of parabola (direction OF) and direction of the light ray. From this equation, it follows \( y = 2\beta \).

The length \( s \) of the parabola segment is calculated as:

\[ s = c \left[ 1 + 2(2h/c)^{3/2} - 2(2h/c)^{5/4} + \ldots \right] \] (3)

\[ s = 4h^2 + c^2/[4h] \]

\[ \ln[(2h + [4h^2 + c^2]/4)\sqrt{2}/(c/2)]^{0.5} \] (4)

The distance \( y \) of the parabolic concentrator is

\[ y = 2f/(1 - \cos \theta). \] (5)

This distance is one of the sides of inlet rectangle aperture area of parabolic concentrator. This area is inlet to solar radiation to the concentrator. Cord \( c \) of the parabola is given by expression

\[ c = 2d = 4f \sin \theta \sin \gamma \] (6)

Height \( m \) of the parabola is given as

\[ m = y \cos 2\beta. \] (7)

The height \( h \) of parabola is given as

\[ h = f + m. \] (8)

Angle \( \theta \) is given as

\[ \tan(\theta) = 2 \sin(2\delta)/(1 + \cos(2\delta)); \] (9)

\[ \theta = \arctan(2 \sin(2\delta)/(1 + \cos(2\delta))). \] (10)

Distance \( x \) is given as

\[ x = h / \cos(\theta); x = d / \sin(\theta). \] (11)

Aperture area of the concentrator that is inlet to solar radiation is given as

\[ S_y = L \times y. \] (12)

Here, \( L \) stands for side length of the concentrator (in east-west direction).

Area \( S_f \) of the absorber to the solar radiation is given as:

\[ S_f = L \times f_p. \] (13)

Here, \( f_p \) stands for the width of solar radiation absorber. In this case \( f_p = f/2 \) where \( f \) is the focal length given in Fig. 4.

As solar absorber has plate shape, the absorber can absorb solar radiation at its front and at its back.

The circumference of the half cylinder at the cylindrical part of the reflector of this concentrator:

\[ C = \pi f/2. \] (14)

Total circumference of the entire reflector is given as:

\[ R = C + s. \] (15)

Total area of aluminum foil of the concentrator is given by expression:

\[ S_R = L \times R. \] (16)

3.2 Geometrical concentration

Two types of the geometrical concentration are defined where one is for heat collection and one is for electricity collection. Geometrical concentration for heat collection is defined by using the following equation

\[ G = y/(f/2) = 2(y/f) \] (17)

Here \( y \) stands for focal length \( f/2 \) is the width of the absorber plate). As \( y = 2f + m \), \( f = (y - m)/2 \) so
\[ G=\frac{4y}{(y-m)}=\frac{4}{1-\cos(2\delta)} \] (18)

Figure 6 shows the geometrical concentration for heat production from solar energy for the concentrators with different aperture angle. Its value can be from 3.6 to 46.

Geometrical concentration for electricity generation is defined by using the following equation

\[ G=\frac{y}{f}=\frac{y}{f} \] (19)

Here, \( y=2f+m \), \( f=(y-m)/2 \) so

\[ G=\frac{2y}{(y-m)}=\frac{2}{1-\cos(2\delta)} \] (20)

Figure 6 shows the geometrical concentration for electricity production from solar energy for the concentrators with different aperture angle. Its value can be from 1.8 to 23.

### 3.3 Revenue from concentrator

The revenue \( P \) [€/year] from concentrator operation is given as

\[ P=P_r+P_e \] (21)

here, \( P=P_r+P_e \) stand for the revenue from heat [€/year] and \( P_e \) stands for the revenue from electricity [€/year].

Direct solar radiation \( Z_{dy} \) [Wh/y] to the concentrator aperture is given as

\[ Z_{dy} = Z_d S_y \] (22)

here, \( Z_d \) [Wh/(m2 y)] stands for the direct solar radiation to the concentrator aperture during year and per m².

The generated heat \( Q \) [kWh/year] at the concentrator absorber

\[ Q=Z_{dy} \eta_t/1000 \] (23)

here, \( \eta_t \) stands for the efficiency of heat generation. It is taken here that \( \eta_t = 0.6 \).

Revenue from generated heat is

\[ P_r=\frac{Q}{100}C_t. \] (24)

here, \( C_t \) [€c/kWh] stands for the price of heat.

The generated electricity \( E \) [kWh/year] at the concentrator absorber

\[ E=Z_{dy} \eta_e/1000 \] (25)

here, \( \eta_e \) stands for the efficiency of electricity generation by absorber. It is taken here that \( \eta_e = 0.15 \).

Revenue from generated electricity is given as

\[ P_e=\frac{E}{100}C_e \] (26)

here, \( C_e \) [€c/kWh] stands for the price of electricity.

### 3.4 Investment in concentrator

The investment in concentrator \( C_U \) [€] consists of the investment in the concentrator absorber \( C_A \) [€], the concentrator reflector \( C_R \) [€], and concentrator construction and assembly \( C_C \) [€]:

\[ C_U=C_A+C_R+C_C. \] (27)

Total cost for aluminum for reflectors is given as:

\[ C_R=\frac{S_\text{R}}{A} \] (28)

here, \( C_A \) [€/m²] stands for the aluminum unit price. Currently in Serbia, \( C_A \approx 9\) Euro/m². Although the reflector consists of other elements this time their price is not taken into account.

Total price of photovoltaic plate is given as:

\[ C_A=S_f C_{F2} \] (29)

here, \( C_{F2} \) [€/m²] stands for the price of the photovoltaic plate per m².

Although the absorber consists of other elements this time their price is not taken into account.

Total price of the concentrator construction and assembly is taken as:

\[ C_C=S_y C_{C2} \] (30)

here, \( C_{C2} \) [€/m²] stands for the price of the location where the concentrator is placed per m².

Although the concentrator construction and assembly consists of other expenses, this time they are not taken into account.

### 3.5 Economy parameters

Payback of investment is given as

\[ R=C_U/P \] (31)

\[ 344 \]
Net revenue during concentrator life cycle is given as:

$$NB = Y \cdot P\cdot CU$$

(32)

Here $Y$ stands for the life of the concentrator in years. Here, discount rate through life of the project was taken to be zero, as there are no valid predictions for Serbia about its value in future.

4. Results and Discussion

In this research, the simulation is performed for seven concentrators that had different aperture angle $\delta$ and the same absorber size. When the meteorological data for Kragujevac are included, then the yearly simulation of the performance of the concentrator will provide us heat and electricity revenue. In addition, the geometrical characteristics of each concentrator will be calculated and corresponding investment costs. These calculations will enable us to obtain the economy of the project: payback of investment and the net revenue during the concentrator life.

4.1 Geometry of concentrator

Figure 5 shows the reflector area of the concentrator as a function of the aperture angle of the concentrator.

4.2 Economy of concentrator

Figure 7 shows the investment and revenue vs. aperture angle. When the revenue is calculated it is taken that the prices for generated electroenergy and heat are 5 €c/kWh. The investment and revenue are lower with higher aperture angle, however decrease in value for the investment is higher than that for the revenue, when aperture angle has lower value.

Figure 7. Investment and revenue vs. aperture angle

Figure 8 gives the payback of investment for the concentrators with different aperture angles.

Figure 8. Payback of investment for the concentrators with different aperture angles

Figure 9 gives the net benefit of the investment for the concentrators with different aperture angles. The concentrator life is assumed to be 15 years. The net benefit decreases for higher aperture angle.

5. Conclusion

Temperature increase due to green-house CO2 emission from different fossil sources is recorded in atmosphere. World community tries to respond to such a situation by increased use of solar energy for heating, cooling, and electricity production.

Figure 9. Net benefit for the concentrators with different aperture angles. The concentrator life is 15 years
In this paper, we present the simplified theory of the solar concentrator that uses solar energy to generate heat and electricity. The concentrator is stationary trough, and has parabolic-cylindrical reflector surface. Its geometrical concentration depends on the concentrator aperture angle and type of energy production and is between 1.8 and 36. For the climate of Kragujevac, Serbia, at 44 deg. north it is found that the revenue, investment and net benefit decrease with aperture angle of the concentrator. However, the concentrator with the minimum payback has 20° aperture angle.

References