

## Reasoning with Linguistic Preferences Using NPN Logic

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**Abstract.** Negative-positive-neutral logic provides an alternative framework for fuzzy cognitive maps development and decision analysis. This paper reviews basic notion of NPN logic and NPN relations and proposes adaptive approach to causality weights assessment. It employs linguistic models of causality weights activated by measurement-based fuzzy cognitive maps' concepts values. These models allow for quasi-dynamical adaptation to the change of concepts values, providing deeper understanding of possible side effects. Since in the real-world environments almost every decision has its consequences, presenting very valuable portion of information upon which we also make our decisions, the knowledge about the side effects enables more reliable decision analysis and directs actions of decision maker.

**Keywords:** Fuzzy Cognitive Maps; Negative-Positive-Neutral Logic; Linguistic Preferences; Decision Analysis.

### 1. Introduction

In the mid-seventies Axelrod [1] proposed representational framework for causal knowledge, namely cognitive maps (CM). Although essentially important this framework has relatively rigid structure. This is due to the fixed ("minus-plus": (-, 0, +) or (-1, 0, +1)) causality measures between concepts. Such structure hardly could capture dynamical behavior of the systems. Further progress in this field provided Kosko in the mid-eighties [2], proposing non-rigid measures of concepts causality. Basic causal relationship between concepts can be described not only as "increasing" or "decreasing", what

“plus” and “minus” means in original cognitive maps setting, but also as “increasing to some degree” and “decreasing to some degree”. These measures can be expressed and assessed by human experts in numbers as well as by words (with deeper meaning), depending on subjective experience, beliefs and practice. In the most cases this kind of measures usually obey certain mathematical laws which classify them as fuzzy measures. Hence, such framework with non-rigid structure for causal knowledge representation is called fuzzy cognitive maps (FCM). These causal schemes, which allow propagation of causality, represent a system behavior in an adaptive way, depicting situations in given environment more realistic.

In general, systems modeling requires high level of expertise to properly identify and represent complex interrelationships between their elements in order to achieve stable operation. Such state represents equilibrium of the system. One of the well known approach to aggregating multicriteria to provide an overall decision function on system behavior includes different classes of ordered weighted aggregation (OWA) operators [8]. However, OWA operators in their definition suggest ordered, permuted position of the weights, instead of direct association with each particular criteria (attribute, system variable). Although reaching equilibrium in the physical system may resemble an aggregation of different criteria, which provides the sort of “optimal balance” of the system, it does not assume an arbitrary layout and interconnection of the system's elements and their corresponding weighted links. In other words, systems are connected sets of elements that behave as a whole, and therefore causally dependent. Furthermore, according to the nature of the physical systems and, consequently, their behavior, the sequence of parameters tuning requires more or less strict interdependent order, which does not assume property of symmetry. Also, the system's components are interlinked, and links represent their weighted dependency, described by so-called connection matrix, where the sum of (all) weights is not necessarily equal to one ( $\sum w_i \neq 1$ ). In such situation FCM's representational framework can provide necessary multicriteria analysis of a (complex) systems.

Since FCMs model systems, we cannot expect that precise tuning of one element will not, in some sense, negatively effect other elements disabling their perfect tuning. That is, we can only approximately or more or less accurately for the best, tune the elements in order to achieve optimal operation of a system as a whole. Therefore, it is of a big importance to measure such effects that “negatively” affect the elements (concepts) of a system, directing our actions toward achieving optimality and balance. By the end of eighties of the last century, Zhang et al. [3] presented generic approach to cognitive map development and decision analysis and introduced negative-positive-neutral (NPN) logic, which provide the basics for reasoning with logic values from [-1, 1] interval and modeling of negative or, so-called, “side effect” of decisions.

In the original setting of FCMs theory it is stated that causal weights between concepts are constant and only the concepts values change in time [2], [4]. This holds even in the case of concepts that are difficult to measure,

such as social, socioeconomic, geographical, medical, military, political, etc. Values of these mostly perception-based concepts also change in time. However, the strength of influence of one concept over another may change in time as well. This fact represents the leading motivation for the research work directed to develop more adaptive framework of FCMs (based on NPN logic). Such more realistic FCM based model of system behavior should include preferential and time-dependent causal weights. For instance, engineering problems usually deal with measurement-based concepts, some of which are time-varying. But independently of their dynamics and direction of causal relationships, intensity of causal dependence may have its own dynamics too. This issue is recognized as the crucial one for adaptive and reliable behavior of decision making [5-8], [27], [29-31], control problems [9], [10] and agent systems [11]. Several approaches are proposed, which include modifications of the Pool2 algorithm [9], [10], [12], neural networks [13-15] and genetic algorithms [13], [16], [17].

This paper presents more adaptive application of FCMs based on NPN logic modeling framework. The approach itself combines NPN logic based FCMs with linguistic models of causality weights to achieve their quasi-dynamical adaptation to the change of measurement-based values of FCM concepts. Next section briefly reviews notion of NPN logic and NPN relations. The third section introduces fuzzy models of FCM causality weights and incorporates them in the framework of NPN logic based reasoning. Finally, a short discussion of presented approach is provided.

## 2. Theory of NPN logic and NPN relations

We recall that FCMs are signed, fuzzy weighted and directed graphs with feedback [2], [4]. The concept nodes  $C_i$  are fuzzy sets or even fuzzy systems. In general, FCM's concept nodes may stand for states, variables, events, actions, goals, values, trends of the system it models. The links, or edges, define rules or causal flows between the concept nodes. The modeling framework is based on determination of meaningful concepts, connecting them to form a network, and evaluating the direction of effect of target concept excited by the cause concept. In such networks (graphs) the directed link (edge)  $w_{ij}$ , from causal concept  $C_i$  to target (effect) concept  $C_j$ , measures how much  $C_i$  causes  $C_j$ . Connection  $n$ -by- $n$  matrix  $W$  contains weights of all edges, representing weighted causation rules of system behavior. The edges  $w_{ij}$  take values in the fuzzy causal interval  $[-1, +1]$ . When FCM models a physical system (for instance, machining process planning, hydroelectric power station, scoliotic deformity, etc.) each concept node is characterized by a number in the interval  $[0, 1]$ . Such concept values are the result either of the normalization of the real value of the system's characteristics or membership degree of the real value (sensory readings, actual measurements) to the fuzzy set which describes the system's characteristics [18]. The latter holds for the concept nodes that represent fuzzy systems, as

well. However, one of the most essential parts in a FCM modeling is determination of causal links between the nodes, including their strengths. One must have in mind that initially set weights, acquired in different ways (e.g. assessed by experts), may change and different weight sets can produce the same equilibrium situation [9], [29].

(F)CMs draw causal relationships between system's nodes, describing their mutual dependability and enabling *what-if* inference on a given situation. When value of one or more concept (reference, input) nodes changes, the map is excited and starts adjusting the values of all other nodes, according to the selected threshold function, until it settles down to equilibrium [4], [9], [10], [19]. This updating process uses both, concept values and causal weights.

In the real-world environments almost every decision has its consequences, presenting very valuable portion of information upon which we also make our decisions. FCM modeling framework involves negative edge weights as well, usually in multivalent [-1, +1] interval. In order to measure the magnitude of consequences we need more than three trivalent interval values {-1, 0, +1} of logic variables [1], [20]. NPN fuzzy logic theory is multi-valued logic based on six classes of values [3], [21]. Three individual classes assume values from [-1,0), {0}, and (0,+1] intervals, and three compound classes of values: (0, P) indicates there is no induced negative relationship and positive relationship has a strength P, (N, 0) indicates there is no induced positive relationship and negative relationship has a strength N, and (N, P) indicates that object *i* has both positive and negative relationships to object *j* with negative relationship of a strength N, and positive relationship of a strength P. The third compound value pair (*a*, *b*) is the most informational and fully describes the side effect, which measures under what mutual conditions between concepts FCM settles down in equilibrium.

Any NPN logic value can be represented as an ordered pair in [-1, 1] × [-1, 1]. The NEG, AND, and OR functions for both NPN crisp and fuzzy logics can be compactly described by the following three logic equations:

$$\text{NEG}(x, y) = (\text{NEG}(y), \text{NEG}(x)) \quad , \quad (1)$$

$$(x, y) * (u, v) = (\min(x * u, x * v, y * u, y * v), \max(x * u, x * v, y * u, y * v)) \quad , \quad (2)$$

$$(x, y) \text{ OR } (u, v) = (\min(x, u), \max(y, v)) \quad . \quad (3)$$

The star operator (\*) in (2) stands for a general conjunction operator that may be any *T*-norm extended from the interval [0, 1] to [-1, 1]. The extension is made as follows:

$$x * y = \text{sign}(x) \text{sign}(y)(|x| * |y|) \quad , \quad (4)$$

where *x* and *y* are singleton NPN values (fuzzy or crisp).

For the sake of briefness we will introduce the following definitions of NPN fuzzy relations, their transitivity and (heuristic) transitive closure, which play important role in reasoning with NPN relations, and skip some other formal definitions, which one can look for in [3], [22-25].

The following definition is an extension of classical fuzzy (binary) relation [22-25], which ensures assigning of NPN compound logic values to a NPN fuzzy (binary) relation as an ordered pair of negative, positive or neutral values:

**Definition:** An NPN fuzzy (binary) relation  $\mathcal{R}$  in  $X \times Y$ , where  $X = \{x_i\}$  and  $Y = \{y_j\}$  are finite sets, is a collection of ordered pairs or a subset of  $X \times Y$  characterized by a membership function  $\mu_{\mathcal{R}}(x_i, y_j)$  that associates with each ordered pair  $(x_i, y_j)$  a strength of relation between  $x_i$  and  $y_j$  using an NPN fuzzy logic value.

One of the very important sources of imprecision in complex systems is related to a transition behavior [4]. The effect of (imprecise) information propagation through a system may have significant influence on final decision-making, depending on weights of connections between concept nodes of a system's network. Next definition provides formal description of max-\* transitivity property of NPN relations:

**Definition:** An NPN relation  $\mathcal{R}$  (crisp or fuzzy) in  $X \times X$ , where  $X = \{x_1, x_2, \dots, x_n\}$  is finite set, is NPN (max-\*) transitive iff, for all  $i, j$ , and  $k$ ,  $0 < i, j, k \leq n$ ,

$$\mu_{\mathcal{R}}(x_i, x_k) \geq \max_{x_j} (\mu_{\mathcal{R}}(x_i, x_j) * \mu_{\mathcal{R}}(x_j, x_k)). \quad (5)$$

Since the connections between system's concepts can be established by different relations we need to compose two or more relations in order to model information propagation, in FCMs usually represented by a fuzzy chain [2], [22-24], [26]. The (max-\*) composition of two NPN relations  $\mathcal{R} \subseteq X \times Y$  and  $\mathcal{Q} \subseteq Y \times Z$ , denoted by  $\mathcal{R} \circ \mathcal{Q}$ , is defined by:

$$\mu_{\mathcal{R} \circ \mathcal{Q}} = \max_y (\mu_{\mathcal{R}}(x, y) * \mu_{\mathcal{Q}}(y, z)), \quad x \in X, y \in Y, z \in Z, \quad (6)$$

and can be extended to n-fold composition denoted as  $\mathcal{R}^n = \mathcal{R} \circ \mathcal{R} \circ \dots \circ \mathcal{R}$ .

**Definition:** The transitive closure  $\tilde{\mathcal{R}}$  of an NPN relation  $\mathcal{R}$  (crisp or fuzzy) in  $X$ , is the smallest (max-\*) transitive NPN relation containing  $\mathcal{R}$ . Since the NPN logics used for transitive closure computation can be considered as a set of rules (heuristics), such closure is called a heuristic transitive closure (HTC) of  $\mathcal{R}$ .

Using an heuristic path searching algorithm [3] we can find the possible and the most effective paths from one concept to another. That means, we can find the paths between elements (concept nodes) of FCM with the strongest negative and positive side effects that constrain decision making, according to the above two definitions.

### 3. Fuzzy Modeling of Causal Weights

When dealing with physical systems concepts' values usually are measurement-based. In certain cases experts rely on their experience and perception of current situation, assessing the concepts' values in imprecise manner (e.g. "approximately 5"). Also, some of precisely measured, obtained or assessed concepts values experts use to convert to less precise classification groups assigning to it descriptive degree of belonging. On the other side, the degrees of causal dependence between concepts in practice are almost with no exceptions qualitatively assessed. The meaning of the words of natural language used for degrees of causal relationships depend on context, domain, and nature of the related concepts. In the most cases just a few words are used to quantify a causal dependence (e.g., weak, medium, strong) and a couple of words for modifiers (e.g., very, extremely, a little, not\_so, fairly).

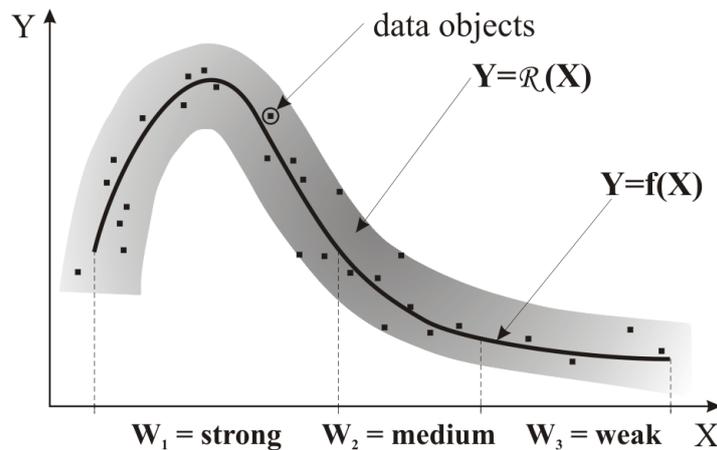
Let  $\mathcal{W}$  be a set of linguistic labels used for quantification of causal weights, namely  $\mathcal{W} = (W_1, W_2, \dots, W_k)$ , defined over domain  $X$ , i.e.,  $W_i \in \mathcal{P}(X)$ ,  $i \in \mathbf{N}_k$ , where  $\mathcal{P}(X)$  denote a power set of  $X$ . Each fuzzy set  $W_i$ ,  $i \in \mathbf{N}_k$  is defined by its membership function  $\mu_{W_i}(x): X \rightarrow [0, 1]$ . Also, let  $\mathcal{M} = (M_1, M_2, \dots, M_r)$  be a set of modifiers, i.e. unary operators (acting on a fuzzy set, transforming a fuzzy set into another one in the same universe), which may modify linguistic weights  $W_i \in \mathcal{P}(X)$ ,  $i \in \mathbf{N}_k$  for each  $x \in X$  by the equation:

$${}^M W(x) = M(W(x)) \quad , \quad (7)$$

where  ${}^M W \in \mathcal{P}(X)$  denotes linguistic value obtained by applying modifier  $M$  to the weight  $W$ . The set of all modified weights is denoted as  ${}^{\mathcal{M}} \mathcal{W}$ . Typically, modifier "very" is defined as  $M(a) = a^2$ , and "a little" or "not\_so" or "fairly" as  $M(a) = \sqrt{a}$ , where  $a \in [0, 1]$ . Of course, for such modifiers we can use other appropriate operators instead. Generally, modifier is called strong if  $M(a) < a$ , weak if  $M(a) > a$ , and identity modifier if  $M(a) = a$ , for  $\forall a \in [0, 1]$ . Thus, each linguistic weight is a subset of  ${}^{\mathcal{M}} \mathcal{W}$ , i.e.,  $\mathcal{W} \subset {}^{\mathcal{M}} \mathcal{W}$ , and has a following structure:

$$\mathcal{S} = \{W, M\} \quad . \quad (8)$$

We may assume that there exist a mapping between values of the concept  $C_i$  and the concept  $C_j$ . It may be a functional dependency between cause concept  $C_i$  and target concept  $C_j$  of the form  $C_j=f(C_i)$ . Function  $f$  may have very complex form, empirically defined or unknown. Also, it may be a relational dependency between cause concept  $C_i$  and target concept  $C_j$  of the form  $C_j=\mathcal{R}(C_i)$ , where relation  $\mathcal{R}$  may be crisp or fuzzy. Experts usually do not provide the definition of such mapping, no matter whether they know it or not. Quite often they rely on recommendation, provided in the form of tables, and to the great extent on their personal experience and belief. Therefore, functional or relational dependence between concepts experts apt to causally describe using linguistic weights. Such linguistic weights are context dependent and often of quasi-dynamic nature. In some cases, while certain value resides within one particular interval of its domain expert will evaluate in one way, but he or she will use different quantifier when concept changes its state to another interval (Fig.1). That is, to achieve the preferred goals of the (physical) system, represented by the equilibrium point (state), causal weights need to be feasibly updated according to the function and operational characteristics of the system.



**Fig. 1.** Possible types of mappings between FCM concept nodes

When deal with measurement-based concept values we usually can gather enough data from direct measurements, archives or recommendation to create data patterns. Also, permissible sets of concepts' values are known. In order to establish fuzzy models of causal weights between concepts we are supposed to cluster given sets of data patterns into classes [25]. The number of classes should be the same as the number of basic linguistic quantifiers which experts use to evaluate system's causal relationships (Fig.1). If needed, basic linguistic quantifiers can be modified by appropriate modifier during any stage of decision analysis process. Thus, the number of clustering classes is  $c = |\mathcal{W}|$ , and the number of possible linguistic labels in a term set

$T = \mathcal{M} \mathcal{W}$  is  $t = |\mathcal{M} \mathcal{W}|$ . Schematically, (type-1) fuzzy model based causal effect of concept node  $C_i$  to concept node  $C_j$  is shown in Figure 2, for both increasing or positive (Fig.2(a)) and decreasing or negative (Fig.2(b)) causal relationship.

Total effect of concept  $C_i$  to concept  $C_j$  is achieved via all paths from  $C_i$  to  $C_j$ . The sequence of concepts  $\mathcal{C} = (C_i, C_1, C_2, \dots, C_j)$  is called chain, namely, NPN fuzzy chain (Fig.3). The strength of the chain is defined by  $*$ -composition of chain elements strength, i.e. by  $*$ -composition of pairs  $(C_k, C_{k+1})$ ,  $k \in \mathbb{N}_{n-1}$ :

$$\mu_{\mathcal{C}}(x) = \mu(C_1, C_2) * \mu(C_2, C_3) * \dots * \mu(C_{k-1}, C_k) \quad (9)$$

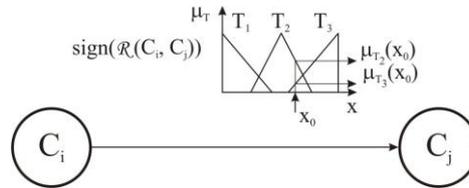


Fig. 2. Fuzzy model based causal weigh

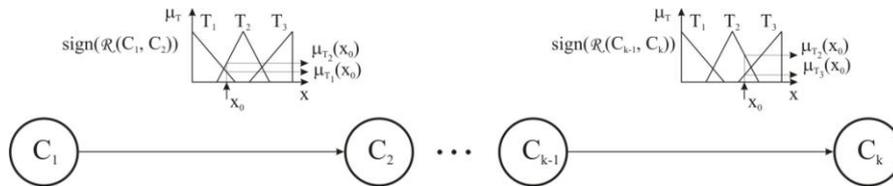


Fig. 3. Fuzzy model based NPN fuzzy chain

Since more than one chain (path) can be established between two nodes in FCM, in general case (Fig.4), for the most important, in the sense of the most causally effective path, we choose the strongest one defined by **max**- $*$  composition (Eq.6).

The causal connection between concepts is established by fuzzy models, which are fuzzy rules. Therefore we apply the same reasoning mechanism as for SISO (single input, single output) systems [25], [28]. Each causal linguistically weighted link is the fuzzy rule of the form:

$$\begin{array}{l} \text{IF} \quad C \text{ is } V \\ \text{THEN} \quad \mathcal{M} \mathcal{W} \text{ is } {}^M W \end{array} \quad (10)$$

where  $C$  is crisp variable represented by a FCM concept defined over domain  $\mathcal{D}_C$ , and  $V$  is a value (current state) of variable and  $V \in \mathcal{D}_C$ ;  $\mathcal{D}_C$  denotes a fuzzified domain obtained upon permissible set of concepts' real values. Rules (10) are relations  $\mathcal{R}_k, k \in \mathcal{N}_q$  in a space  $\mathcal{D}_C \times X$ , defined as:

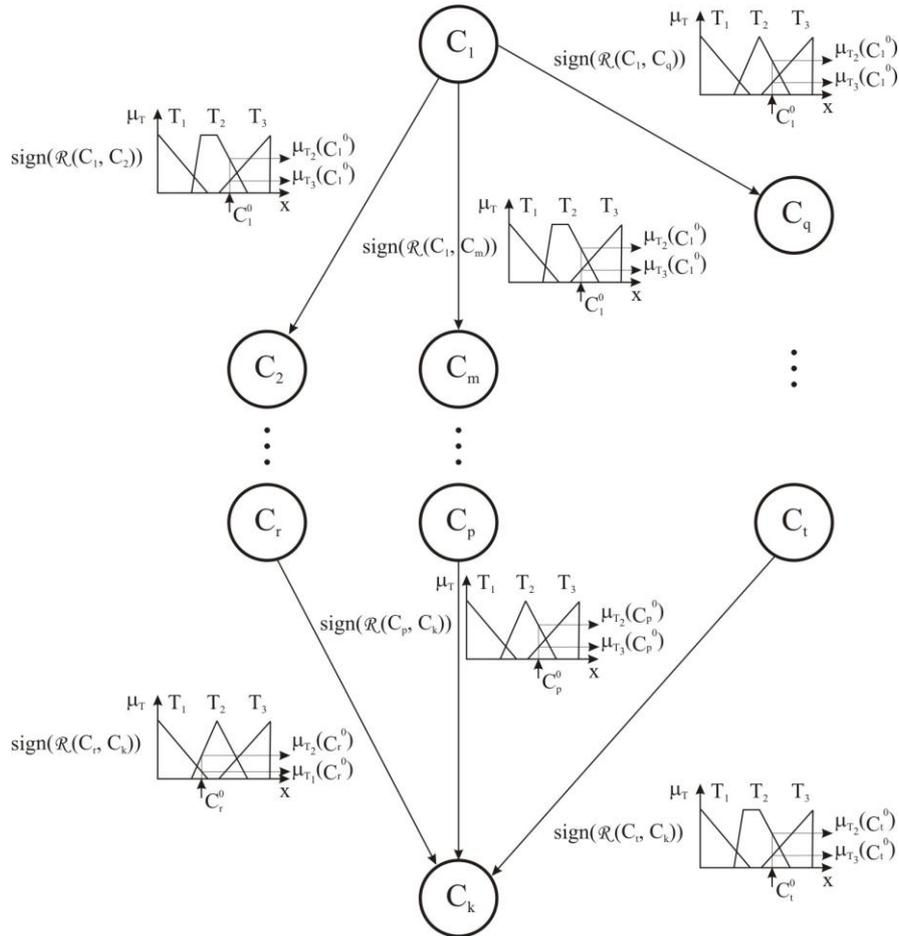


Fig. 4. Fuzzy model based NPN fuzzy multiple chains

$$\mathcal{R}_k(\mathcal{D}_C, X) = \{(V_k, {}^M W_k), V_k \in \mathcal{D}_C, {}^M W_k \in \mathcal{M} \mathcal{W}\}, \quad (11)$$

and membership function

$$\mu_{\mathcal{R}_k}(v, x) = M(\mu_{W_k}(v, x)) \quad , \quad (12)$$

defines a strength of causal link, where  $M$  is modifying unary operator.

The strength of the NPN fuzzy chain (individual causal path  $P$ ) defined by Eq.(9) is  $*$ -composition of partial relations (11):

$$\mathfrak{R}^{(P)} = \underset{k=1}{*}^q \mathfrak{R}_k, \quad (13)$$

and corresponding membership function is:

$$\mu_{\mathfrak{R}^{(P)}}(C_i, C_j) = \underset{t}{*}^t (\mu_{\mathfrak{R}_k}(C_i, C_t), \mu_{\mathfrak{R}_k}(C_t, C_j)) . \quad (14)$$

The global relation  $\mathfrak{R}$  represents the strongest individual causal path of concept  $C_i$  to concept  $C_j$  defined by **max**-composition (denoted by  $\cup$ ):

$$\mathfrak{R} = \underset{j=1}{\cup}^r \mathfrak{R}_j^{(P)}, \quad (15)$$

of the total strength:

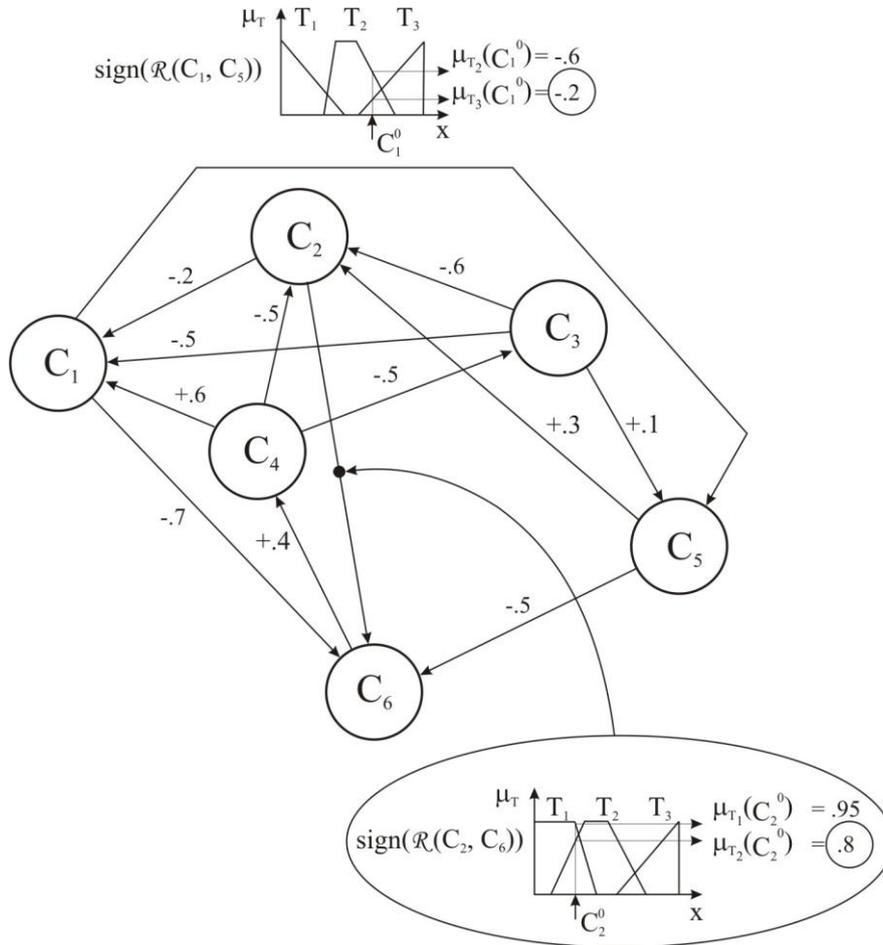
$$\mu_{\mathfrak{R}}(C_i, C_j) = \underset{k=1}{\cup}^m \mu_{\mathfrak{R}_k^{(P)}}(C_i, C_j) . \quad (16)$$

As the result of fuzzy relation  $\mathfrak{R}$  we obtain the lower and the upper bound value of NPN logic value pair  $(a, b)$ , which count side-effect that defines conditions of equilibrium in a system.

### **Illustrative example**

Let's suppose that a physical system is modeled by the FCM shown in Figure 5. Its concept nodes represent distinctive measurement-based characteristics. Also, mutual dependence between two causal nodes may be of different nature, assuming that representational schemes include functional, relational and empirical modes. Consequently, fuzzification algorithms may vary, respecting the basic requirement to produce a term set of a linguistic causal weights, in this case  $\mathcal{W} = (Low, Medium, High)$ . We adopt the following set of modifiers:  $\mathfrak{M} = (Very, Fairly, \mathcal{I})$ , with usual definitions for modifiers *Very* and *Fairly*, where  $Very(a) = a^2$  and  $Fairly(a) = \sqrt{a}$ ,  $a \in [0,1]$ , and  $\mathcal{I}$  represents identity modifier  $\mathcal{I}(a) = a$ , for  $\forall a \in [0,1]$ . The corresponding term set is  $T = \mathfrak{M} \mathcal{W}$ . For the sake of simplicity, without

the loss of generality, the most of the causal weights in the Figure 5 are shown as resulting membership grades, rather than fuzzy models.



**Fig. 5.** NPN FCM with linguistic causal weights assessment

Also, let concept node  $C_1$  be a reference (input) node, and concept node  $C_2$  be an output node. Assuming that we are interesting in making a decision on control action over concept node  $C_2$  when excite concept node  $C_1$ , we start "what-if" analysis by identifying the fuzzy chains which describe the possible paths from  $C_1$  to  $C_2$  using heuristic path searching algorithm [3]. Among all these paths we calculate the most effective one that provides the largest side effect by applying Eqs.(11)-(16). For the given physical system corresponding connection matrix is:

In this case we have chosen *max-prod* (max-dot) transitivity composition for the output node, defined by Eqs. (5) and (6). Identified the most effective

heuristic paths are shown in the Table 1 and the compound values of heuristic transitive *max-prod* closure are shown in the Table 2. If we are interested how to most effectively increase the value (i.e. to perform a control or decision action) of concept node  $C_2$ , which can be thought as increasing the speed or temperature, then we conclude that the most effective is the path 1-6-4-2, which provides the highest positive compound value 0.140, with the strongest side effect -0.084, caused through the chain 1-6-4-3-2.

$$W = \begin{pmatrix} 0 & 0 & 0 & 0 & -0.2 & -0.7 \\ -0.2 & 0 & 0 & 0 & 0 & 0.8 \\ -0.5 & -0.6 & 0 & 0 & 0.1 & 0 \\ 0.6 & -0.5 & -0.5 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 & 0 & -0.5 \\ 0 & 0 & 0 & 0.4 & 0 & 0 \end{pmatrix} \quad (17)$$

The change of the initial concept nodes' values, affects causal weights to modify according to their mutual physical dependency. Such changes may generate new causal weights using the approach described above. That is, connection matrix (17) changes using the set of modifiers  $\mathcal{M}$ , as presented by matrix (18). The most effective heuristic paths changed, as well as the compound values of heuristic transitive *max-prod* closure (Tables 3 and 4).

$$W = \begin{pmatrix} 0 & 0 & 0 & 0 & -0.2 & -Fairly(0.7) = -0.84 \\ -Very(0.2) = -0.04 & 0 & 0 & 0 & 0 & 0.8 \\ -0.5 & -Very(0.6) = -0.36 & 0 & 0 & 0.1 & 0 \\ 0.6 & -0.5 & -0.5 & 0 & 0 & 0 \\ 0 & Fairly(0.3) = 0.55 & 0 & 0 & 0 & -Fairly(0.5) = -0.71 \\ 0 & 0 & 0 & Fairly(0.4) = 0.63 & 0 & 0 \end{pmatrix} \quad (18)$$

Now we can notice that performing a control or decision action over the concept node  $C_2$  is again the most effective through the path 1-6-4-2, but with increased overall benefits. The positive compound value is increased for almost 90% and reached 0.265, while the side effect, increased for about 31% at level of -0.110, is caused through the another path, 1-5-2. In the first setting of the hypothetical physical system the desired effect is caused before the side effect, since the side effect chain is longer, but in the modified setting the side effect is caused before the desired effect, for the same reason.

Table 1: The most effective heuristic paths (initial system's setting)

	1			2			3			4			5			6		
	N	P		N	P		N	P		N	P		N	P		N	P	
1	1-6-4-1	1-6-4-2-6-4-1		1-6-4-3-2	1-6-4-2		1-6-4-2-6-4-3	1-6-4-3		1-6-4	1-5-6-4		1-5	1-6-4-3-5		1-6	1-5-6	
2	2-1	2-6-4-1		2-1-6-4-2	2-1-6-4-3-2		2-6-4-3	2-1-5-6-4-3		2-1-5-6-4	2-6-4		2-6-4-3-5	2-1-5		2-1-5-6	2-6	
3	3-2-1	3-1		3-2	3-5-2		3-1-6-4-3	3-1-6-4-2-6-4-3		3-1-5-6-4	3-1-6-4		3-1-6-4-1-5	3-5		3-5-6	3-1-6	
4	4-3-2-1	4-1		4-2	4-3-2		4-3	-		4-1-6-4	4-1-5-6-4		4-1-5	4-3-2-1-5		4-1-6	4-1-5-6	
5	5-2-1	5-2-6-4-1		5-6-4-3-2	5-2		5-2-6-4-3	5-6-4-3		5-6-4	5-2-6-4		5-2-6-4-1-5	5-6-4-1-5		5-6	5-2-6	
6	6-4-3-2-1	6-4-1		6-4-2	6-4-3-2		6-4-3	-		-	6-4		6-4-1-5	6-4-3-2-1-5		6-4-1-6	6-4-1-5-6	

Table 2: Compound values of heuristic transitive max - prod closure (initial system's setting)

	1			2			3			4			5			6		
	N	P		N	P		N	P		N	P		N	P		N	P	
1	-0.168	0.027		-0.084	0.140		-0.022	0.140		-0.280	0.040		-0.200	0.014		-0.700	0.100	
2	-0.200	0.192		-0.028	0.017		-0.160	0.004		-0.008	0.320		-0.016	0.040		-0.020	0.800	
3	-0.500	0.120		-0.600	0.030		-0.070	0.011		-0.020	0.140		-0.017	0.100		-0.050	0.350	
4	-0.060	0.600		-0.500	0.300		-0.500	-		-0.168	0.024		-0.120	0.012		-0.420	0.060	
5	-0.060	0.058		-0.060	0.300		-0.048	0.100		-0.200	0.096		-0.012	0.024		-0.500	0.240	
6	-0.024	0.240		-0.200	0.120		-0.200	-		-	0.400		-0.048	0.005		-0.168	0.024	

**Table 3:** The most effective heuristic paths (modified system's setting)

	1		2		3		4		5		6	
	N	P	N	P	N	P	N	P	N	P	N	P
1	1-6-4-1	1-6-4-2-6-4-1	1-5-2	1-6-4-2	1-6-4-2-6-4-3	1-6-4-3	1-6-4	1-5-6-4	1-5	1-6-4-3-5	1-6	1-5-6
2	2-1	2-6-4-1	2-1-6-4-2	2-1-6-4-3-2	2-6-4-3	2-1-5-6-4-3	2-1-5-6-4	2-6-4	2-6-4-3-5	2-1-5	2-1-5-6	2-6
3	3-2-1	3-5-2-6-4-1	3-2	3-5-2	3-1-6-4-3	3-1-6-4-2-6-4-3	3-1-5-6-4	3-1-6-4	3-1-6-4-1-5	3-5	3-5-6	3-1-6
4	4-3-2-1	4-1	4-2	4-3-2	4-3	-	4-1-6-4	4-1-5-6-4	4-1-5	4-3-2-1-5	4-1-6	4-1-5-6
5	5-2-1	5-2-6-4-1	5-6-4-3-2	5-2	5-2-6-4-3	5-6-4-3	5-6-4	5-2-6-4	5-2-6-4-1-5	5-6-4-1-5	5-6	5-2-6
6	6-4-3-2-1	6-4-1	6-4-2	6-4-3-2	6-4-3	-	-	6-4	6-4-1-5	6-4-3-2-1-5	6-4-1-6	6-4-1-5-6

**Table 4:** Compound values of heuristic transitive max - prod closure (modified system's setting)

	1		2		3		4		5		6	
	N	P	N	P	N	P	N	P	N	P	N	P
1	-0.318	0.080	-0.110	0.265	-0.067	0.265	-0.529	0.089	-0.200	0.026	-0.840	0.142
2	-0.040	0.302	-0.011	0.004	-0.252	0.002	-0.004	0.504	-0.025	0.008	-0.006	0.800
3	-0.500	0.017	-0.360	0.055	-0.132	0.033	-0.045	0.265	-0.032	0.100	-0.071	0.420
4	-0.007	0.600	-0.500	0.180	-0.500	-	-0.318	0.054	-0.120	0.001	-0.504	0.065
5	-0.022	0.166	-0.081	0.550	-0.139	0.224	-0.447	0.277	-0.033	0.054	-0.710	0.440
6	-0.005	0.378	-0.315	0.113	-0.315	-	-	0.630	-0.076	0.001	-0.318	0.054

## 4. Conclusions

We have presented some of preliminary results of the research work related to the adaptive causality weights assessment. It is based on application of linguistic preferences and their fuzzy models. In this approach system behavior analysis uses linguistic causality weights constructed upon measurement-based concepts values and their known dependences. This way causality weights are assessed indirectly, activating fuzzy models by current state concepts values. In this approach the nature of concept dependency is captured by clustering measurement-based data into appropriate groups. To each group we assign a label of expert's linguistic preference used to describe the system behavior. Linguistic models quasi-dynamically tune the causation weights and allow for their propagation through NPN logic fuzzy chains. The lower and upper bound of NPN logic compound values enable measurement of side effect, which in turn provides the basis for deeper understanding of system behavior and more reliable decision analysis.

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