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DETERMINATION OF GEROTOR PUMP THEORETICAL FLOW

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Summary: The methodology for developing the mathematical expressions for determination of gerotor pump theoretical flow is presented in this paper. The theoretical flow is equal to current alteration of volume at working chambers that are linked with output side of the pump. For the calculation of the working chamber volume at gerotor pump the method that consider the influence of infinity small rotation angles of pump's working elements to elementary changing of volume are used and determination of working volume of chamber is done after integration. The developed analytical model is illustrated by numerical examples in which the influences of design parameters to variations of gerotor pump flow are analysed. The pump with fixed axes of shafts is considered in this paper but developed expressions by establishing specific kinematic relations can be, also, used at pumps with planetary motions.

Key words: trochoidal gearing, positive displacement pumps, gerotor pumps, volume of working chamber, volumetric flow

1. INTRODUCTION

Gerotor is the mechanism with internal trochoidal gearing that was realized by Myron F. Hill at 1906. The name GEROTOR is derived from the phrase GEnerated ROTOR and described mathematical procedure for generating peritrochoid profile of inner gear by circular arc of the external profile. Gerotor can be used in cases where gear pumps with external gearing are present and, also, it can be used where gear pumps with internal or fixed displacement vane pump are present: for cooling and lubricating systems, so as for transfer of liquids.

Gerotor pumps belong to the group of rotational pumps and they have great advantages in relation to other types of rotational pumps. Some of the advantages are simple constructions and variety of applications. Due to specific geometry of gears

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profiles, continual contacts of all teeth are provided in exploitation that obtains the necessary separation between the low and high pressure zones. During the operation, teeth of the pump rotor act as pistons while chambers (the space between profiles of inner and gears) correspond to cylinders. According to the presented facts, the current volumes of chambers increase and decrease periodically while those chambers are connected alternately to input and output line. The single alteration of the current volume of the chamber from minimal to maximal value is indicated as one operation cycle. Due to specific construction of the pump, several operation cycles are done during one rotation of the shaft.

In order to obtain the high functional characteristics of the pump it is necessary to analyse the influences of numerous of different parameters to its output characteristics of the pump. According to that, the basis of the investigation presented in this paper is identification of influence of the geometric parameters variations to volumetric characteristics by contemporary analysis based on modelling and simulation.

2. MODELLING OF TROCHOIDAL GEARING AT THE GEROTOR PUMP

In this paper, gearing at the gerotor pump with profile of internal gear is determined by equidistant of the peritrochoid while external profile is determined by circular arc with radius r_c is considered.

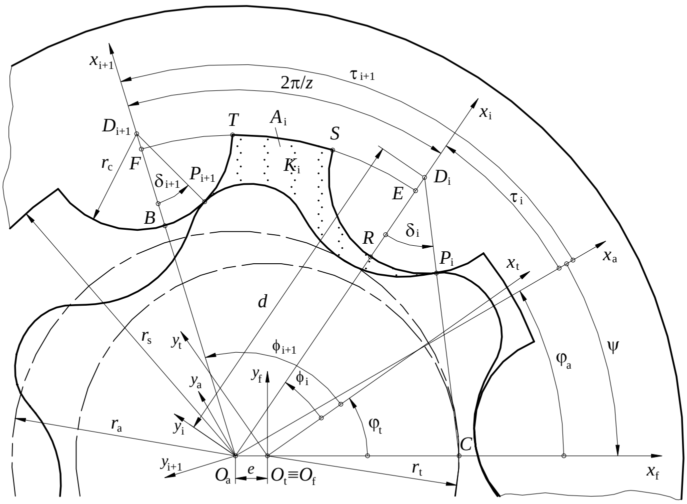


Fig. 1 *Schematic presentation of the gerotor pump gear pair with basic geometrical dimensions*

The basic geometric relations for the generation of peritrochoid, at the considered gear pump, are shown at Fig. 1, that are adopted for defining the relations to determine the current position of contact point P_i in coordinate system $O_a x_a y_a z_a$ in following form [7]:

$$\mathbf{r}_{P_i}^{(a)} = \begin{bmatrix} e\left[z\lambda\cos\tau_i - c\cos(\tau_i + \delta_i)\right] \\ e\left[z\lambda\sin\tau_i - c\sin(\tau_i + \delta_i)\right] \\ 1 \end{bmatrix}, \tag{1}$$

where is: z - number of the external gear teeth, e - eccentricity (center distance between the internal and external gear), λ - trochoid coefficient, $\lambda = d/ez$, d distance between the generating point D and the center of the external gear, c equidistant coefficient, $c=r_c/e$, τ_i – angle that determined the position of the point D_i, $\tau_i = \dfrac{\pi(2i-1)}{z}$, $i = 1, \ldots, z$, δ - leaning angle [2-5]. For determination of angle δ the following relation is used:

$$\delta_i = \arctan \frac{\sin(\tau_i - \psi)}{\lambda - \cos(\tau_i - \psi)}, \tag{2}$$

where ψ is referent rotation angle.

3. DETERMINATION OF THE PUMP THEORETICAL FLOW

Instantaneous flow rate of the pump presents the volume variation of working chambers in time and can be calculated by following relation [1]:

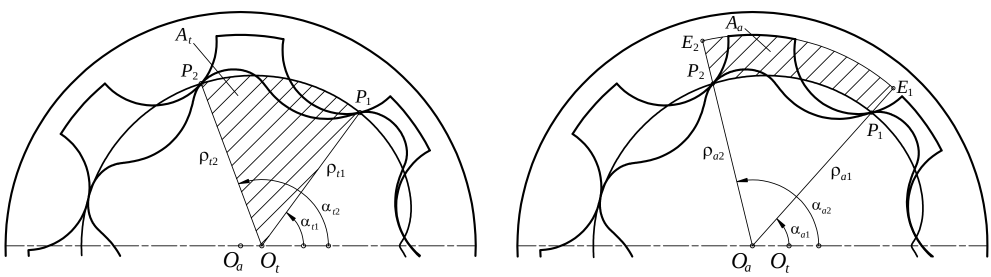
$$Q = \sum_{i=m}^{n} \frac{dV_i}{dt}, \tag{3}$$

where is: V_i - current volume of considered working chamber, m, n - indexes of the beginning and final chambers which can be found at the same time in the thrust phase.

For calculation of the volume of the gerotor pump working chamber, the method that consider the analysis of influence of infinitesimal values of rotation angles of the pump's working elements is used. The volume of working chamber is calculated by integration. For calculation of the variation of the pump's working chamber, the equivalent system with fixed axis of gear pair elements is considered. The relation between the rotation angles of working elements is determined in following form $d\varphi_t = \dfrac{z}{z-1} d\varphi_a$. The variation of working chamber volume is analysed by the area of segments with form of circular fragment. In general cause, the area of A segment that are bordered with curve determined in polar coordinate system and the position vectors of points that correspond to polar angles α_1 and α_2 is calculated by the following formula [8]:

$$A = \frac{1}{2} \int_{\alpha_1}^{\alpha_2} \rho^2 d\alpha, \tag{4}$$

where ρ and α are polar coordinates of presented curve.



(a) *(b)*

Fig. 2 *Presentation of segments that are considered for determination the variation of working chamber volume*

Firstly, the variation of volume V_t that is pushed by internal gear would be analysed. This volume is bordered, at cross section, by radius nominated as ρ_{t1} and ρ_{t2} of the consequent contact points P_1 and P_2 and by circular segment between those points (Fig. 2 (*a*)). During rotation of internal gear for angle of $d\varphi_t$, radius ρ_{t1} rotate for polar angle of $d\alpha_{t1}$ and by that, it push the volume of $\frac{1}{2}b\rho_{t1}^2 d\alpha_{t1}$, while the alteration in the length of the radius is neglected because the considered values are infinitesimal (Fig. 3). As internal gear rotates faster than the point on contact line, relate to $d\varphi_t > d\alpha_{t1}$, the formed volume is $\frac{1}{2}b\rho_{t1}^2 d\mu_{t1}$, where is $d\mu_{t1} = d\varphi_t - d\alpha_{t1}$. According to that, the variation of volume for contact point P_1 will be equivalent to $\frac{1}{2}b\rho_{t1}^2(d\alpha_{t1} + d\mu_{t1}) = \frac{1}{2}b\rho_{t1}^2 d\varphi_t$. When the same analysis is done for contact point P_2, the relation for calculation of volume variation of internal area of considered chamber can be formed as:

$$dV_{t1} = \frac{1}{2}b\left(\rho_{t2}^2 - \rho_{t1}^2\right)d\varphi_t\,. \tag{5}$$

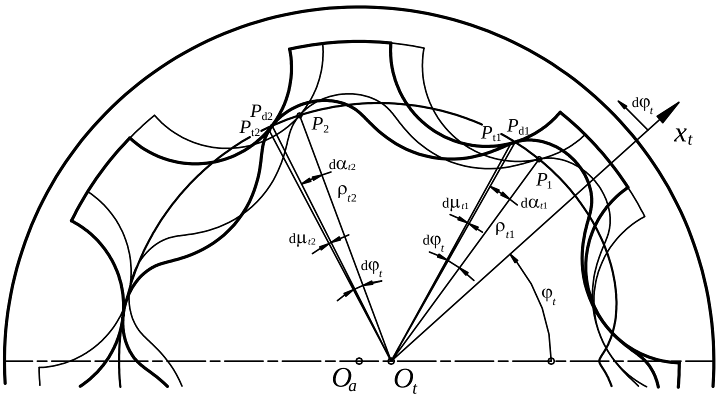


Fig. 3 *Volume variation of internal area of working chamber*

Now, by the similar method, the volume variation V_a that is pushed by external gear can be calculated. This volume is bordered, at cross section, by linear segments formed by extending of radius ρ_{a1} and ρ_{a2} of the consequent contact points P_1 and P_2 and with circular segment of contact line between mentioned points and circular arc E_1E_2 (Fig. 2 (*b*)). During rotation of external gear for angle of $d\varphi_a$, radius ρ_{a1} rotate for the polar angle of the $d\alpha_{a1}$ and by that push the volume of $\frac{1}{2}b\left(r_s^2 - \rho_{a1}^2\right)d\alpha_{a1}$ (Fig. 4). As external gear rotate slower than point on contact line, relate to $d\varphi_a < d\alpha_{a1}$, the formed volume would be $\frac{1}{2}b\rho_{a1}^2 d\mu_{a1}$, where is $d\mu_{a1} = d\alpha_{a1} - d\varphi_a$. According to this, the variation of external volume for contact point P_1 will be equivalent to $\frac{1}{2}b\left(r_s^2 - \rho_{a1}^2\right)d\alpha_{a1} - \frac{1}{2}b\left(r_s^2 - \rho_{a1}^2\right)d\mu_{a1} = \frac{1}{2}b\left(r_s^2 - \rho_{a1}^2\right)d\varphi_a$. When the same analysis is done for the contact point P_2, the relation for calculation of volume variation of external area of considered chamber can be formed as:

$$dV_{a1} = \frac{1}{2} b\left(\rho_{a1}^2 - \rho_{a2}^2\right) d\varphi_a . \tag{6}$$

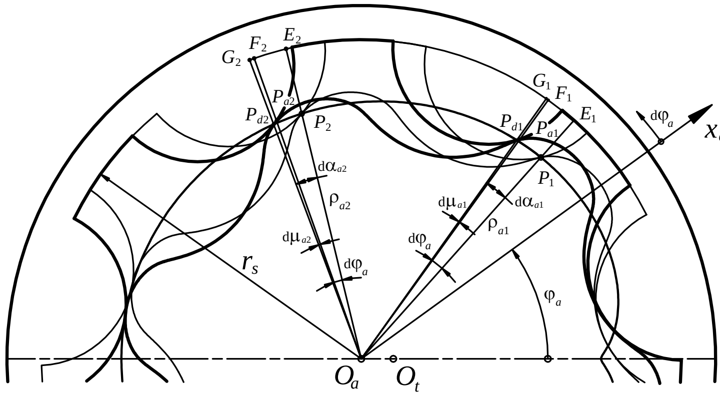


Fig. 4 *Volume variation of external area of working chamber*

The result volume variation of working chamber is equivalent to:

$$dV_1 = dV_{t1} + dV_{a1} . \tag{7}$$

Starting from equitation (5) and (6) it is implicate that:

$$dV_1 = \frac{1}{2} b\left[\left(\rho_{a1}^2 - \rho_{a2}^2\right) d\varphi_a + \left(\rho_{t2}^2 - \rho_{t1}^2\right) d\varphi_t\right], \tag{8}$$

and taking into account the relation between the rotation angles of gears, following form is obtained:

$$\frac{dV_1}{d\psi} = -\frac{1}{2} b\left[\rho_{a1}^2 - \rho_{a2}^2 + \frac{z}{z-1}\left(\rho_{t2}^2 - \rho_{t1}^2\right)\right] \tag{9}$$

and after that, it is transformed into the form suitable for determination of current geometrical flow:

$$\frac{dV_1}{dt} = \frac{1}{2z} b(z-1)\omega_t \left[\rho_{a1}^2 - \rho_{a2}^2 + \frac{z}{z-1}\left(\rho_{t2}^2 - \rho_{t1}^2\right)\right]. \tag{10}$$

In order to obtain the simple form of final relations for integration for determination of current flow according to equitation (3), the following relation can be established:

$$\left(CP_2\right)^2 - \left(CP_1\right)^2 = (z-1)\left[\rho_{a1}^2 - \rho_{a2}^2 + \frac{z}{z-1}\left(\rho_{t2}^2 - \rho_{t1}^2\right)\right]. \tag{11}$$

When equitation (11) compare to equitation (9), the general relation for determination of chamber's volume variation can be written in the form:

$$\frac{dV_i}{d\psi} = -\frac{b}{2(z-1)}\left[\left(CP_{i+1}\right)^2 - \left(CP_i\right)^2\right], \; i = 1, \; \ldots \; , z. \tag{12}$$

The distance CP_i of contact point P_i from kinematic pole C is determined as intensity of vector $\mathbf{CP}_i^{(a)}$. Radius vector of the point C in the frame $O_a x_a y_a z_a$ could be written in the form

$$\mathbf{r}_C^{(a)} = \begin{bmatrix} ez\cos\psi \\ ez\sin\psi \\ 1 \end{bmatrix} = ez\cos\psi\,\mathbf{i}_a + ez\sin\psi\,\mathbf{j}_a\,. \tag{13}$$

Vector $\mathbf{CP}_i^{(a)}$ based on geometrical relations from the Fig. 1 and equation (1) and (13), can be expressed as

$$\mathbf{CP}_i^{(a)} = \mathbf{r}_{P_i}^{(a)} - \mathbf{r}_C^{(a)} = \begin{bmatrix} e\left[z\lambda\cos\tau_i - z\cos\psi - c\cos(\tau_i+\delta_i)\right] \\ e\left[z\lambda\sin\tau_i - z\sin\psi - c\sin(\tau_i+\delta_i)\right] \\ 0 \end{bmatrix}, \tag{14}$$

so it is:

$$(CP_i)^2 = \left|\mathbf{CP}_i^{(a)}\right|^2 = \left[x_{CP_i}^{(a)}\right]^2 + \left[y_{CP_i}^{(a)}\right]^2 = e^2\left\{z\left[1+\lambda^2-2\lambda\cos(\tau_i-\psi)\right]^{\frac{1}{2}} - c\right\}^2. \tag{15}$$

On the basis of the obtained relation with using of certain trigonometric transformations, the following formula for determination of theoretical flow variation in chamber K_i is formed as:

$$\frac{dV_i}{dt} = \omega_t b e^2 z\left\{2\lambda\sin\frac{\pi}{z}\sin\left(\frac{2\pi i}{z}-\psi\right) - \frac{c}{z}\left[1+\lambda^2-2\lambda\cos(\tau-\psi)\right]^{\frac{1}{2}}\Bigg|_{\tau_i}^{\tau_{i+1}}\right\}. \tag{16}$$

4. TESTING OF THE MATHEMATICAL MODEL OF THE PUMP FLOW

The influence of different parameters to volume variation would be illustrated on numerical models. Two different gear pairs are considered (Fig. 5), the commercial one with mark GP-575, and other one gear pair, GP-850, with profile formed on the results of calculations presented in the paper [7]. Geometrical parameters of the considered gear sets are: gear set **I** z=6, λ=1.575 and c=3.95, gear set **II** z=5, λ=1.850 and c=3.95. The remaining parameters are: e=3.56 mm, r_s=26.94 mm, b=16.46 mm.

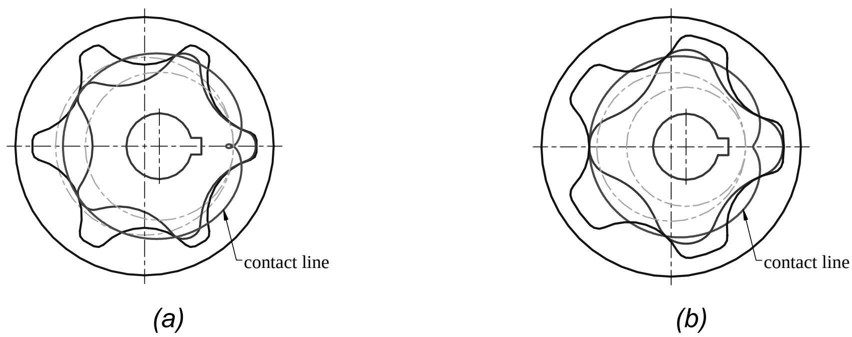
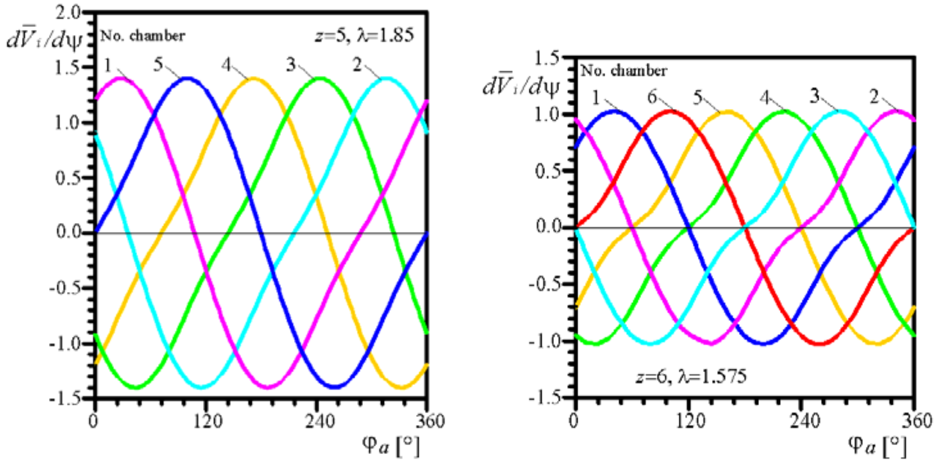


Fig. 5 *Schematic presentation of the considered trochoidal pump models (a) GP-575 (commercial) and (b) GP-850*

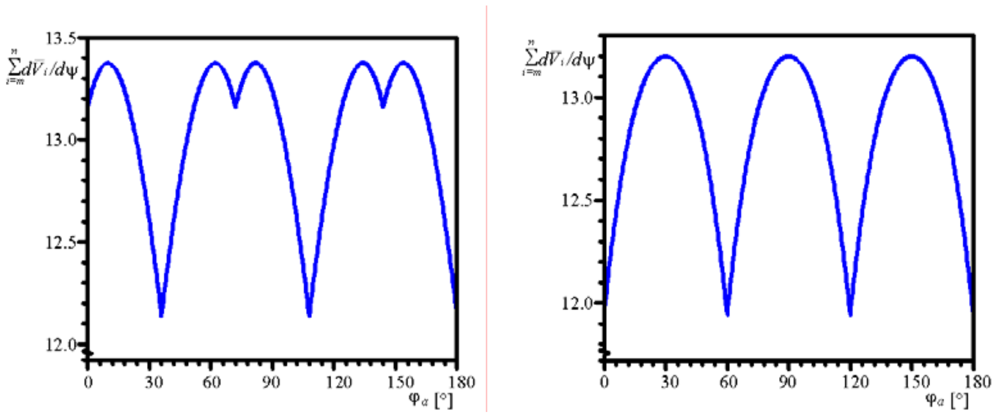
By using of the computer program developed on the basis of the mathematical model of the pump geometrical flow, for the given parameters of the gear pump the results are obtained that are presented by graphics at the following figures. The fluctuations of flow rate of the pump for different chambers are presented at Fig. 6. On the basis of the presented fluctuations the conclusions about value and variation of the current flow can be done. For the graphical interpretation the following dimensionless parameter is induced $\overline{V}_i = \dfrac{V_i}{e^2\, b}$.



(a) *(b)*

Fig. 6 *Diagrams of flow rate variation at different chambers of the pump with five (a) and six (b) chambers*

On the graph of the variation of current flow during ending of thrust and at the beginning of the suction phase the local extreme is present. At the pump with five chambers the variations are smaller. Diagrams of current flow pulsations at the pump with different number of teeth in relation to rotation angle of external gear are presented at Fig. 7.



(a) *(b)*

Fig. 7 *Diagrams of instantaneous flow rate at the pump with five (a) and six (b) chambers*

As measure of flow uneven the flow rate irregularity δ_q is induced that characterize the relation between current flow to average value of the flow [6]. For the pump with six chambers flow rate irregularity is equivalent to $\delta_q \approx 10$ %. For the pump with five chambers flow rate irregularity is equivalent to $\delta_q \approx 9.7$ % that conformed the fact that pumps with even number of chambers have bigger pulsations, so designs of the pumps with odd number of working chambers are recommended.

5. CONCLUSION

In order to obtain the functional relations that would provide design of the pumps on the basis of the given basic functional requirements, the mathematical model of volumetric characteristics of the pump with trochoidal gearing is developed. The mathematical model was tested and the results are analysed, so the relevant characteristics are identified that influent to pulsation of pump flow and to variation for the flow. The general conclusion is implicated that in order to reduce the pulsations of flow it is recommended to use the odd number of chambers at the pumps.

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