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THE INFLUENCE OF DESIGN MODIFICATIONS OF CARDAN SHAFT DRIVEN FORK ON ITS STRESS DISTRIBUTION

Abstract: *This paper presents analysis of effect of geometric parameters variation to stress level and its distribution at driven fork of Cardan joint in power transmitters. In the study, presented in this paper, the maximal stress at the driven fork is calculated by analytical method, and then, obtained results are compered to numerically obtain one. It is implicated that small geometry modifications of driven fork lead to significant reduction of maximal stress.*

Keywords: *Cardan shaft, Driven Fork, Critical Stress, Numerical Methods, Geometric Parameters*

1. INTRODUCTION

The main function of power transmitter with Cardan shafts is to transmit power and torque between the elements of power group which are on certain distances and that are not rigidly connected. They can be used between elements without coincide in axes or with variations in relative positions. The Cardan joint get its name after Italian mathematician Girolami Cardano, who invented this type of joint. The Cardan shaft with two joints is usually used. This type of shaft provides flexibility of positions to the angle of 30° between of axes. Usage of Cardan shafts in power transmitters of cars and trucks, building and agricultural machinery is essential [1].

The identification of Cardan shafts and its elements are based on nominal power and torque. But, Cardan shaft, selected by this method, do not always satisfied the requirements for reliability and duration of exploitation period.

The causes of failures and design of power transmitters with Cardan shafts are analyzed by many researches. Hummel and Chassapis [2] researched on the design of the universal joints. They have given some suggestions on the configuration design and optimization of universal joints with manufacturing tolerances [3]. Bayrakceken *et al.* [4] performed the fracture analysis of a universal joint fork and a drive shaft of an automobile power transmission system. Spectroscopic analyses, metallographic analyses and hardness measurements are carried out for each part of power transmitter with Cardan joint.

In most cases, the major influential factor to dimensions and reliability of Cardan shaft are the properties of its fork. In the study presented in this paper, the stress limit at the fork is calculated by analytical method, and then, obtained result is compared to numerically obtain one. Finite elements method is used to analyze the stress distribution in critical zone. On the bases of the obtained results the optimization of fork design was done.

2. CARDAN JOINT DRIVEN FORK

Cardan shafts are systems made up of of elements, arranged and linked together. A system is characterized in that it can be delineated from its surroundings, with the links to the surroundings at the boundary of the system. A variety of factors affects the selection of the sub-systems. The elements of the Cardan shaft systems are shown at Figure 1.

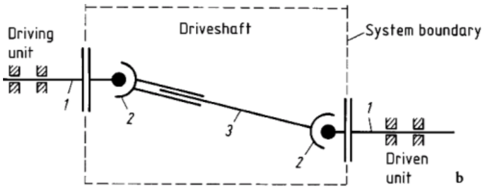

Figure 1. Scheme of Cardan shaft elements: 1 - connecting shaft, 2 - joints and 3 - intermediate shaft

Cardan joint fork, in dependence of type of connection with adjoining shaft can be equipped with bearing or flange. The flange couple the fork of the Cardan joint with second shaft. The coupling can be obtained also by groove joint or by welding. There are different designs of the Cardan joint fork with different types of joining to adjoining shaft. The difference between them are mainly in method of joining to shaft and in the form of bearings for the cross shaft. Scheme of the concidered fork is presented at Figure 2.

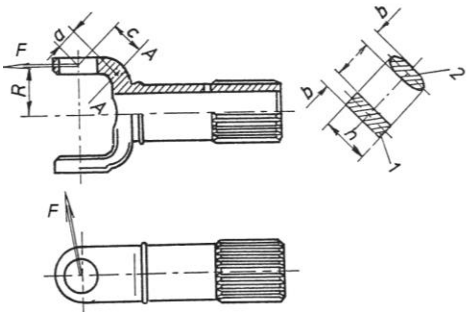

Figure 2. Scheme of Cardan joint fork [5]

2.1 Stress calculation at Cardan joint driven fork by analytical method

Calculation of Cardan joint fork is done on the basis of maximal torque and maximal force due to transmission of nominal power [5-6]. Analytical and numerical calculation of stresses at Cardan joint driven fork was carried out with its real shape and dimensions. After that, the model parameters are changed and the stress distribution was calculated by numerical method in order to identify the best design solution with the lowest level of stresses [7]. The basic load data, material properties and dimensions of fork for calculation are given at Table 1.

Table 1. Basic data of the considered Cardan joint

Name	Value
Input power	P_U = 10 *kW*
Input number of rotations	n_1 = 1500 min^{-1}
Angle of path	α_{12} = 30°
Shear modulus	$G = 0.8 \cdot 10^5$ *MPa*
Dimensions of driven fork	R = 15.5 *mm*
	h = 32 *mm*
	b = 13.5 *mm*
	c = 35 *mm*
	a = 19 *mm*

On the basis of nominal parametars and load configuration, the calculation of maximal torque of T_{u2max} = 77.775 *Nm* was done, so as the calculation of maximal load force F = 2.509 *kN* [7-11].

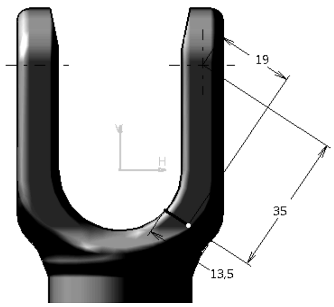

Figure 3. Cardan joint driven fork

Cardan joint driven fork is loaded to bending and torsion. Bending stresses in the base of the Cardan joint fork sleeve (critical section A-A at Figure 2), can be determined as follows [5]. Rotary momentum of inertia W_b and W_s depend on shape of cross section of branches of Cardan joint fork. For adopted ellipsoid shape of branchs of driven fork, after certain assumptions, following relation can be used:

- Resistant bending torque

$$W_b = \frac{bh^2}{10} = 1.382 \cdot 10^{-6} m^3 \qquad (1)$$

- Resistant shear torque

$$W_s = \frac{1}{5} hb^2 = 3.902 \cdot 10^{-7} m^3 \qquad (2)$$

- The bending stress

$$\sigma = \frac{F \cdot c}{W_b} = 63.52 MPa \qquad (3)$$

- The shear stress

$$\tau = \frac{F \cdot a}{W_s} = 40.87 MPa \qquad (4)$$

- The equivalent stress

$$\sigma_e = \sqrt{\sigma^2 + 3\tau^2} = 95.11 MPa \qquad (5)$$

The stress is less than the allowable maximal stress limit that is adopted as: σ_d = 150 *MPa* [5].

2.2 Structural analysis of driven fork by finite element method

Structural analysis by finite element method, made by CATIA® software is done and obtained results are put in correlations to values of stresses obtained analytically for the driven fork. The comparisons are done to evaluate the relevances of the results. On the basis of the identificated relevant results, the effect of changes of the shape to the maximum stress was analyzed.

Fork of the Cardan joint are under torque load at both sides with same intensity, so one half of the fork can be considered. The modifications of the design solution can be done at this considered half due to the symmetry of element.

The stress distributions for four different design solutions of Cardan joint driven fork were analyzed. Firstly, the three dimensional tetrahedron discretizations with variation of dimensions of elements are used. The transition zone between fork and cylindrical part was discretized by finest elements (2 *mm* in dimension at the edge) while the rest of the analyzed element was discretized by the elements with dimension of 2 x 5 *mm*. The boundary conditions were defined according to theoretical considerations. The load was set to force of 2509 *N* on the branch of the fork for every case of design solution. The numerical model of Cardan driven fork was completely defined and statically determined.

Case 1: Geometrical model was simplified in this case without profiling the branchs of driven fork (Figure 4). The stress distribution for this case is presented at Fig. 5. Simplified model is different from the real model due to neglecting of fillet, while dimension of the branches of Cardan joint fork is the same at whole its length. The simplified model of the considered fork is presented at Figure 5. Also, stress distribution at considered model is presented at same figure.

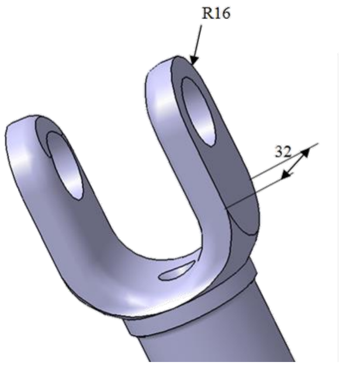

Figure 4. Simplified model of driven fork

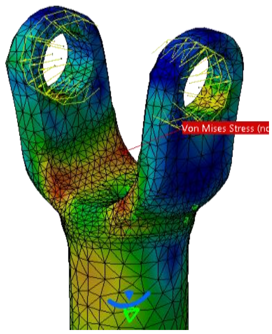

Figure 5. Finite element stress analysis of Cardan joint driven fork (Case 1)

Maximal stress at the base is 90.876 *MPa* at simplified model of the considered Card joint fork. The difference to the analytically obtained value is 4.5%.

Case 2: Real Cardan joint fork is different to simplified one due to fact that dimension of cross section of the branches of the fork is not constant along its length. The dimensions at cross section are reduced at the end of the branches and edges are filleted. Model of the Cardan joint fork is presented at Figure 6. The stress distribution is presented at Figure 7.

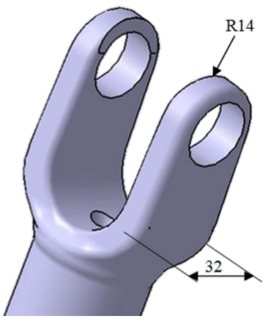

Figure. 6. Model of real driven fork (Case 2)

Maximal stress is 160.523 *MPa* at the point where force acts at the model of the real Cardan joint fork and it is 77% higher then maximal model due to the fact that at dimensions of the cross section are smallest at zone of maximal stress. At the basis of the fork stress is 100 *MPa*, and it is different to analytical value by around 5%. The next step is moving of the point where maximal stress act and to reduce its value.

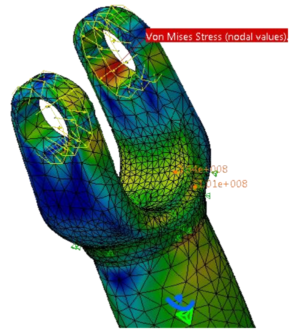

Figure 7. Finite element stress analysis of Cardan joint driven fork (Case 2)

Case 3: If modification to simplified model is done in that way that dimensions of the cross section of the branches of Cardan joint fork at the end become higher, so dimensions of the cross section become constant along branchs length and filleted ends with radius that is equivalent to one half of the dimension at cross section (Figure 8) the better stress distribution is obtained (Figure 9).

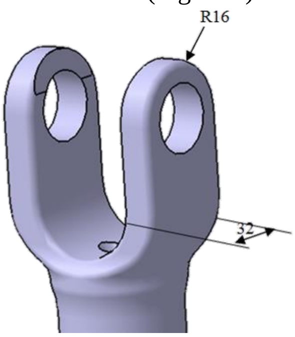

Figure 8. Model of driven fork (Case 3)

Constant dimension of cross section at branches of the fork have significant influence to value of maximal stress due to moving of the zone with stress concentrations from the point of force acting to the base of the branches. The value of the maximal stress is 90.672 *MPa* that is 44% less than maximal stress at the model of the real fork and with 5%

difference to analytically obtained value. The maximal stress acting zone is moved and its value is reduced in relation to value in case 2.

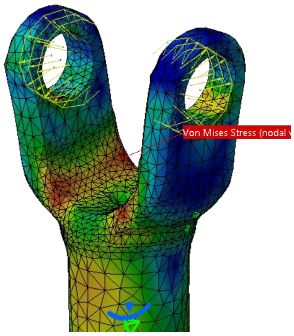

Figure 9. Finite element stress analysis of Cardan joint driven fork (Case 3)

Case 4: If enlargement of the cross section at the branches have positive influence to reduction of the stresses that in this case at the end of the branches dimension rises from 32 *mm* to 36 *mm* (Figure 10).

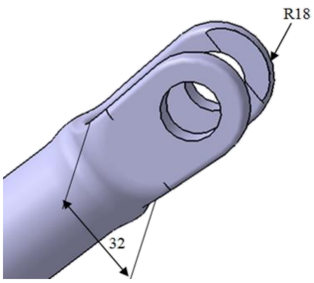

Figure 10. Model of the driven fork (Case 4)

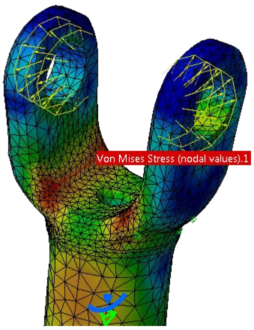

Figure 11. Finite element stress analysis of the Cardan joint driven fork (Case 4)

Obtained stress distribution in this case is presented at Fig. 11. In this case maximal stress at the base is 83.404 *MPa* that is 8% less than in case 3 and with 12% difference to analytically obtained value, so this modification of design is beneficial in relation to other considered cases.

3. RESULTS EVALUATION

On the bases of the results obtained by analysis of Cardan joint driven fork, presented in the paper, identification of the best design solution with lowest level of stresses was done. Variations of maximal stresses at driven fork are given at Tab. 2. Case with lowest level of stresses is identified and chosen as optimal design solution.

Table 2. Variation of maximal stresses at driven fork

Maximal stress calculated by analytical method: 95.11 *MPa*			
No.	Maximal stress	Variation of maximal stress in relation to previous model [%]	Difference to analytic maximal stress [%]
1	90.88		4.45
2	160.52	-76.64	-68.78
3	90.67	43.51	4.67
4	83.40	8.02	12.31

The sign minus represents that the variation of maximal stress in relation to maximal stress obtained in previous model are negative. So, the negative variation means that maximal stress is increased to previous maximal stress. The same manner of notation is used to indicate the variation of maximal stress obtained by numerical calculation to maximal stress calculated by analytical method.

On the bases of the presented analysis the identification of the optimal design of Cardan driven fork was done. The model with optimal design is the one with best design solution with dimension of 36 *mm* of the cross section at the end of the fork branches, while the rest of dimensions remain the same as in the real Cardan driven fork.

4. CONCLUSION

The changing of the maximal stresses at considered driven fork of the Cardan joint is caused by changing of fillet at its ends, so as, by changing of the dimensions at cross section of its branches. The value of maximal stress can be reduced by 12% due to different design solutions. On the basis of the analisis it is concluded that best design solution is one with dimension of 36 *mm* of the cross section at the end of the fork branches (Case 4). In case that this design solution cause increase difficulties at production process, exclusion of hole at the central zone of the fork between of its branches can be done. This modification of design and variation of geometrical parameters of the considered fork in relation to its effects on value of maximal stresses will be subject of future considerations.

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