Disturbance Invariant Speed Controlled Servo Drive with Tesla’s Induction Motor

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Abstract - In his patent [1], [2] "Electro-Magnetic Motor" of May 1, 1888, Tesla wrote: "Be it known that I, Nikola Tesla, a subject of the Emperor of Austria, from Smiljan, Lika, border country of Austria-Hungary, now residing in New York, in the country and State of New York, has invented certain new Improvements in Electro-Magnetic Motors, of which the following is a specification ..." Tesla continued writing about the mode of operation, principles of construction, and advantages of the alternating current motor called today as the induction or asynchronous motor. Several original patents (see, for example, [3]) for Tesla's induction motor, polyphase system for the transmission of the electrical power, and a further series of over 40 patents within the same field from 1887 to 1891 make consistent Tesla's system of the generation, transmission, and various ways of application of alternative electric energy.

From the late nineteenth century to these days, Tesla's motor was advantageously used in many applications where the torque capabilities, acceleration characteristics, power density, and high degree of reliability are indispensable. Nevertheless, in the design of high performance electrical drives, the choice of DC motor was preferable, since its torque and flux are controlled independently through two separate terminals. To match this suitable feature of DC motor in servo applications, the concept of vector control [4], [5] was developed to enable decoupled control of the flux and torque of induction motor; thus, Tesla's motor may enable control characteristics of DC motor [6].

In this paper, it is shown how Tesla's induction motor can be applied in the design of speed digitally controlled servo drive in the presence of immeasurable arbitrary torque disturbances. For rejection of effects of torque disturbance on the steady-state value of motor speed the principle of absorption [7], [8] is applied in the design of disturbance estimator within the control portion of the system controlling structure.

Keywords - Speed controlled servo, Tesla's motor, Absorption principle, Immeasurable disturbance, IMPACT structure.

I. INTRODUCTION

The main task in control system design is the tracking of a reference signal with sufficiently small or zero steady-state error in the presence of unknown external disturbance. In many applications, the designer knows an amount of apriori information about the class of disturbances and limits of interval changes or uncertainties of plant parameters. It is known that the knowledge about mismatches of plant parameters is necessary for the robustness analysis in the design of control systems with internal models [11], [12]. Most frequently an external disturbance may be modeled as a solution of homogenous differential or difference equation. In such case, it is possible to include the disturbance model into the control portion of the system in order to extract completely influence of disturbance on the steady-state value of system output [11]. Essentially, this design procedure is based upon the application of the IMP (Internal Model Principle).

The idea of absorption principle has been primarily presented in the seminal work of Kulebakin [8] who examined the invariance conditions and introduced the concept of so-called selective invariance in the control system theory. Johnson [9] used this principle in the servomechanism design. Davison [10] applied a similar approach of using the internal disturbance model for the rejection of disturbance. This method is based on so-called IMP (Internal Model Principle). Solving the regulator problem in multivariable control systems, Francis and Wonham [11], [12] came to a conclusion that for disturbance rejection it was necessary to include the disturbance model into the control part of the system. As a matter of fact, IPM was inspired by results of works by Francis and Wonham [11], [12]. Utilizing results of Kulebakin [8], Tsypkin and Holmberg [16] expressed concisely the principle of absorption for digital control systems. At the same time, they defined the generalized disturbance and examined the possibility of its extraction [7]. The absorption principle has been advantageously applied in the design of a robust Smith predictor for a process with long dead time [15]. The IMP and principle of absorption are based upon the same fundamental idea of inclusion disturbance model into the system controlling structure.

In this paper, the principle of absorption is formulated in a discrete form and then the possibility of its application in the design of disturbance invariant servomechanisms with Tesla's induction motor is examined. To enable extraction of an apriori unknown disturbance, the suitable modification of IMPACT (Internal Model Principle and Control Together) structure with the adaptation mechanism within the internal model of disturbance is proposed. The ability of the structure to absorb fast disturbances is approved by simulation. Although the idea of adaptation of the disturbance internal model is not the novel one [17], it is first time applied for the design of concrete control plant subjected to fast disturbances. The robust properties of the structure will not be analyzed separately; however, it will be shown how the desired robustness and filtering properties of the system can be achieved by tuning of single parameter in the system controlling structure.
II. PRINCIPLE OF ABSORPTION

Suppose that kth sample of external disturbance \( f(t) \) may be determined by finite number \( m_0 \) of previous samples. Then, the disturbance is regular and may be described by extrapolation equation [7]

\[
f(kT) = D_f(z^{-1})f((k-1)T)
\]

(1)

where \( D_f(z^{-1}) \) is the prediction polynomial of order \( m_0 \). Relation (1) is called the equation of extrapolation or prediction polynomial [7] and it may be rewritten as

\[
(1-z^{-1}D_f(z^{-1}))f(z^{-1}) = 0
\]

(2)

where \( f(z^{-1}) \) denotes the \( z \)-transform of disturbance. Relation (2) is called compensation equation and FIR filter having the pulse transfer function \( (1-z^{-1}D_f(z^{-1})) \) is the absorption filter or the compensation polynomial [7].

Absorption filter \( \Phi_f(z^{-1}) = 1 - z^{-1}D_f(z^{-1}) \) is designed for a known class of disturbances and its impulse response becomes identically equal to zero after \( n \) sampling instants, where \( n \geq m_0 \). Hence, the compensation equation (2) may be considered as the absorption condition of a given class of disturbances. The condition can be expressed as

\[
\Phi_f(z^{-1})f(z^{-1}) = 0, \text{ for } t = kT \geq (\deg \Phi_f)T.
\]

(3)

The extrapolation polynomial \( D_f(z^{-1}) \) is determined by an apriori information about disturbance \( f(t) \) [7], [16]; nevertheless, it is simply resolved from as

\[
\Phi_f(z^{-1}) = w_{\text{den}}(z^{-1}), \text{ from } f(z^{-1}) = \frac{w_{\text{num}}(z^{-1})}{w_{\text{den}}(z^{-1})}.
\]

(4)

In the case of a stochastic disturbance \( s(t) \), absorption filter (4) should suppress as much as possible affects of disturbance on the system output. Thus, for a low frequency disturbance \( s(t) \), which can be generated by double integration of the white noise, an appropriate choice of absorption filter is \( \Phi_s(z^{-1}) = (1-z^{-1})^2 \) that corresponds to absorption of linear (ramp) disturbance [16].

In [7], the absorption principle is generalized to enable the extraction of parameter disturbances. Namely, changes of system output due to differences between the real and nominal plants may be treated as a parameter disturbance \( \varphi_p \), that influences the system output. Thus the generalized system disturbance \( \Phi = \varphi_p + w \) comprises the external disturbances and uncertainties of plant parameters. Hence, the real plant may be considered as the nominal plant subjected to the generalized disturbance \( \Phi \). If the system is robustly stable, the parameter disturbance is regular and its absorption is possible, i.e.,

\[
\Phi_p(z^{-1})\varphi_p(z^{-1}) = 0, \text{ for } t = kT \geq (\deg \Phi_p)T
\]

(5)

with

\[
\Phi_p(z^{-1}) = 1 - z^{-1}D_p(z^{-1})
\]

(6)

where \( \Phi_p(z^{-1}) \) and \( D_p(z^{-1}) \) denote the compensation and prediction polynomials of \( \varphi_p \), respectively.

According to (3) and (5), the absorption condition of generalized disturbance \( \varphi = \varphi_p + f \) becomes

\[
\Phi_f(z^{-1})\varphi(z) = 0, \text{ for } t = kT \geq (\deg \Phi_f + \deg \Phi_p)T
\]

(7)
or

\[
\Phi(z^{-1})\varphi(z) = 0, \text{ for } t = kT \geq (\deg \Phi)T
\]

(8)

where \( \Phi(z^{-1}) \) denotes discrete absorption filter of generalized disturbance.

The design of absorption filter greatly depends upon the amount of apriori information about the disturbance. If the class of disturbance is known in advance (constant, slow varying, ramp, sinusoidal with known radial frequency, etc), the absorption filter can be designed exactly; otherwise, the design of absorption filter becomes more difficult. If the class of disturbance is unknown, the disturbance is to be predicted by evaluating of disturbance signal in each sampling instant. Namely, the coefficient vector \( \mathbf{\tilde{\theta}} \) in prediction polynomial \( D(z^{-1}, \mathbf{\tilde{\theta}}) \) is to be fitted by the on-line identification mechanism based on estimated disturbance [11]. Consequently, the fitting of prediction polynomial coefficients requires the on-line estimation of disturbance. In the case of IMPACT control structure, the information about the generalized disturbance is obtained by including the nominal plant model into the control portion of the system.

In a general case, let \( e(kT) \) be an estimated value of disturbance in sampling instant \( kT \). Then, analogously to (1), the following prediction relation is valid

\[
e(kT) = D(z^{-1}, \mathbf{\tilde{\theta}})e((k-1)T).
\]

(9)

Relation (9) may be rewritten as vector equation

\[
e(kT) = \mathbf{\tilde{\theta}}^{\intercal} \mathbf{\psi}_e(kT)
\]

(10)

where

\[
\mathbf{\psi}_e(kT) = [e((k-1)T), e((k-2)T), ..., e((k-m)T)]^{\intercal}.
\]

(11)

The recurrence algorithm for estimation of coefficient vector \( \mathbf{\tilde{\theta}} \) is given as

\[
\mathbf{\tilde{\theta}}(kT) = \mathbf{\tilde{\theta}}((k-1)T) + \Delta \mathbf{\tilde{\theta}}(kT),
\]

(12)

\[
\Delta \mathbf{\tilde{\theta}}(kT) = \mathbf{\Gamma}_e(kT)(\mathbf{e}(kT) - \mathbf{\tilde{e}}((k-1)T)\mathbf{\psi}_e(kT))\mathbf{\psi}_e(kT)
\]

(13)

where \( \mathbf{\Gamma}_e(kT) \) is the gain matrix, which can be calculated in different ways [11]; the simplest one is based on the Kaczmarz algorithm

\[
\mathbf{\Gamma}_e(kT) = [\mathbf{\psi}_e^{\intercal}(kT)\mathbf{\psi}_e(kT)]^{-1}\mathbf{I}
\]

(14)

Indeed, the adaptation mechanism should be chosen carefully not to be too complicated. In majority of practical applications an appropriate choice might be \( D(z^{-1}) = 2 - z^{-1} \).
According to (4), this prediction polynomial rejects ramp disturbances; but, it enables also the extraction of slow varying disturbances and even it suppresses effects of low frequency stochastic disturbances on the system output, especially if the sampling period \( T \) is chosen sufficiently small. It is to be noted, in the presence of adaptation mechanism, the system behavior becomes slightly nonlinear. Therefore, before the identification of plant parameters, the prediction polynomial should be reduced, as much as possible, into an appropriate form. For example, in the case of a slow varying disturbance \( f(t) \), for which the first difference \( \Delta f(kT) = f(kT) - f((k-1)T) \) is small, an adequate prediction polynomial might be the simplest one \( D_1(z^{-1}) = 1 \).

However, if the first difference is not sufficiently small, but if the quantity \( \Delta f(kT) - \alpha \cdot \Delta f((k-1)T) \), which for \( \alpha = 1 \) becomes equal to the second difference, is small, then the appropriate prediction polynomial

\[
D_j(z^{-1}) = (1 + \alpha) - \alpha \cdot z^{-1} \tag{14}
\]

may be adopted. Then the fitting of adaptation loop is performed by tuning parameter \( \alpha \) in (14).

The successful implementation of the absorption principle greatly depends upon the quality of disturbance model or of amount of a priori information about the disturbance and, in connection with that, on the adequate choice of prediction polynomial. Unlike other controlling structures that utilize the absorption principle, Tsypkin's IMPACT structure implements the absorption filter so that the requirements for an accurate disturbance compensation and system stability conditions are not opposite to each other even in the case of unstable control plants.

### III. IMPACT STRUCTURE

Fig. 1 shows the IMPACT structure proposed by Ya.Z. Tsypkin [7], [16]. Actually, the structure in Fig. 1 represents the modified primarily IMPACT structure suited for speed controlled servo drive with Tesla's induction motor. The control portion of the structure is encircled by dotted lines.

![IMPACT structure of digital control system](image)

The nominal model of the plant comprising the induction motor, vector-controlled inverter, and shaft encoder may be approximated at the low frequency band as

\[
W_o(z^{-1}) = \omega_0(z^{-1}) = \frac{\omega_0(z^{-1})}{u(z^{-1})} = \frac{K_m}{J} \frac{z^{-1}}{1 - z^{-1}} = C_m \frac{z^{-1}}{1 - z^{-1}} \tag{15}
\]

where \( K_m, J, \) and \( T \) denote the electromagnetic torque coefficient, motor inertia, and sampling period, respectively. It is supposed that the pulse transfer function of the real plant may be represented as

\[
W(z^{-1}) = W_o(z^{-1})(1 + \omega W(z^{-1})) \tag{16}
\]

with the known nominal plant model

\[
W_o(z^{-1}) = \frac{z^{-1} P_o(z^{-1})}{Q_o(z^{-1})} = \frac{C_m z^{-1}}{1 - z^{-1}}, \quad C_m = \frac{K_m}{J} \tag{17}
\]

and its perturbation \( \omega W(z^{-1}) \) limited by the multiplicative bound of uncertainties

\[
\left| \omega W(z^{-1}) \right| \leq \alpha(\omega), \omega \in [0, \pi/T] \tag{18}
\]

Parameters of the nominal plant model (17) determine polynomials \( z^{-1} P_0(z^{-1}) \) and \( Q_o(z^{-1}) \) of the two-input internal plant model of the IMPACT structure in Fig. 1. The output \( \epsilon \) of internal plant model estimates the influence of external disturbance on the controlled variable \( \omega \) (shaft speed) [7], [16]. Consequently, signal \( u_1 \) is a portion of manipulated variable \( u \) that should compensate effects of disturbance. In a particular case, when the plant doesn't include a dead time, the internal model of disturbance may be imbedded into an adequate prediction polynomial \( D(z^{-1}) \). In addition, if the minimally phase plant is under consideration, then, according to the standard procedure of IMPACT structure design, one may assume

\[
R(z^{-1}) = P_o(z^{-1}) = C_m \tag{19}
\]

Polynomials \( P_o(z^{-1}) \) and \( P_o(z^{-1}) \) of the main control loop are determined by the specified closed-loop system pole spectrum or by the desired closed-loop system transfer function. Since the closed-loop system is of second order, the stability margin and speed of system set-point response may be specified by the relative damping coefficient \( \varsigma \) and undamped natural frequency \( \omega_o \) of the dominant complex-conjugate pole pair inside the Nyquist frequency band. In doing so, one obtains the desired closed-loop system transfer function as

\[
G_{de}(z^{-1}) = \frac{(1 - (z_1 + z_2) + z_1 z_2) z^{-2}}{1 - (z_1 + z_2) z^{-1} + z_1 z_2 z^{-2}} \tag{20}
\]

where

\[
z_{1/2} = \exp(s_{1/2}T) \quad \text{with} \quad s_{1/2} = -\varsigma \omega_o \pm j \omega_o \sqrt{1 - \varsigma^2} \tag{21}
\]

From (20), Polynomials \( P_o(z^{-1}) \) and \( P_o(z^{-1}) \) becomes

\[
P_o(z^{-1}) = (1 - (z_1 + z_2) + z_1 z_2) z^{-2} \tag{22}
\]

\[
P_o(z^{-1}) = 1 - (z_1 + z_2) + z_1 z_2 z^{-1} \tag{22}
\]

The choice of \( \varsigma \) and \( \omega_o \) significantly affects the dynamic properties of closed-loop system, robust stability, and filtering properties of the system. Note that the robust stability condition plays an important role in the design of control system with internal models. Namely, if this condition is not
satisfied, it doesn’t make sense to discuss the system ability to extract external disturbances. The design of system with prescribed degree of robustness with respect to uncertainties of plant parameters has been outlined in [15]. In the absence of sufficient apriori information about the generalized disturbance, the on-line identification of disturbance and an appropriate adaptation of prediction polynomial coefficients are necessary. In a general case, regardless of the applied adaptation mechanism, the IMPACT structure of the speed controlled servomechanism with asynchronous motor has the structure shown in Fig. 2.

According to (9) and in virtue of Fig. 2, it is evident that signal $\hat{\epsilon}$ estimates the portion of manipulated variable $u$ which should compensate effects of generalized disturbance. In other words, if the system absorbs the disturbance completely, the steady-state value of the error of disturbance estimation

$$
\hat{\xi}(z^{-1}) = e(z^{-1}) - \tilde{\hat{\epsilon}}(z^{-1})
$$

becomes equal to zero. This fact enables the use of an adaptive approach based upon the minimization of $|\hat{\xi}|$. Thus, coordinates $\hat{\theta}$ of vector $\hat{\theta} = [\theta_{1}, \theta_{2}, ..., \theta_{n}]^{T}$ are variable prediction polynomial coefficients that are to be determined by minimizing the performance index

$$
J(\hat{\theta}) = J(\hat{\theta}, \hat{\theta}, ..., \hat{\theta}_{n}) = \frac{1}{2} \hat{\xi}(kT)^{T} \hat{\xi}(kT)
$$

using the gradient method given in [18]. A according to [18], the general recurrence formula of the on-line iterative process of adaptation is

$$
\hat{\theta}(kT + T) = \hat{\theta}(kT) + \gamma \cdot \Delta \hat{\theta}(kT)
$$

(25)

where $\Delta \hat{\theta}(kT)$, in the first place, determines the direction of a change of parameter $\hat{\theta}$ at the sampling instant $kT$,

$$
\Delta \hat{\theta}_{i}(kT) = \frac{\partial J(\hat{\theta}, kT)}{\partial \hat{\theta}_{i}(kT)} = \frac{\partial \hat{\xi}(kT)}{\partial \hat{\theta}_{i}(kT)} = \hat{\xi}(kT) \tilde{\epsilon}(kT)z^{-i}
$$

while the positive scalar $\gamma$ regulates the step of iteration. Namely, the process of iteration converges to the proper parameter vector if the constant $\gamma$ satisfies condition

$$
0 < \gamma < \gamma_{c}
$$

(27)

where $\gamma_{c}$ is the upper limit of convergence step [18]. Practically, the positive constant $\gamma$ is chosen by chance and then the convergence of iterative process is tested [18].

![Fig. 2. IMPACT structure with adaptation of coefficients of prediction polynomial](image)

![Fig. 3. IMPACT structure with predictive filter and adaptation of prediction polynomial](image)

To investigate the ability of controlling structure of Fig. 2 and in absorption of disturbance and to test the efficiency of the proposed adaptation mechanism, several simulation runs were done. The parameters of Tesla’s induction motor used in the speed controlled servo were taken from [6] as the total moment of inertia $J = 0.0459$ kgm, electromagnetic torque constant $K_{m} = 0.01154$, and sampling period $T = 10$. The synthetic plant parameter is calculated as $C_{m} = 2.5 \cdot 10^{-7}$. The desired dominant pole pair of the closed-loop system is specified by $\zeta = 0.82$ and $\omega_{n} = 350$ rad/s. In all simulation runs, the same constant reference signal $\omega_{r}(t) = 0.5 \cdot h(t - 0.1)$ and load torque disturbance $T_{l}(t) = 1.5 \sin(46t) \cdot h(t - 0.25)$, shown in Fig. 4, are applied simultaneously.

IV. ILLUSTRATIVE EXAMPLE

To enable as fast as possible converge, the initial guess of coefficient vector of prediction polynomial $D(z^{-1})$ should be adopted in accordance with an available airport information about the disturbance.

Fig. 3 shows the controlling structure obtained by modifying the structure of Fig. 2 by introducing the simple prediction filter [19]. In the structure of Fig. 3, by decreasing of tuning parameter $c_{p}$ one can enlarged the region of system robust stability and improve filtering properties of the system. At the same time, the decrease of parameter $c_{p}$ slightly derogates the speed (or dynamic) of disturbance absorption.
Fig. 5 shows the transient responses of the system in Fig. 1. Traces $\omega_1(t)$ and $\omega_2(t)$ in Fig. 1 are obtained for $D(z^{-1}) = 1$ and $D(z^{-1}) = 2 - z^{-1}$, respectively. In virtue of Fig. 5, the suppression of load torque disturbances is more efficient when the absorption polynomial $D(z^{-1}) = 2 - z^{-1}$ that corresponds to ramp disturbances is employed than in the case where $D(z^{-1}) = 1$ corresponding to constant disturbances is used. Nevertheless, in both cases the disturbance of Fig. 4 affects the steady-state value of the motor speed, i.e., the complete rejection of disturbance is not achieved.

Fig. 6 visualizes the operation of the system in Fig. 2 in which the proposed adaptation mechanism with $\gamma = 5$ is applied. The trace $\omega_2(t)$ is obtained with the initial polynomial $D_z z(t) = -11$, which converges to

$$D(z^{-1}) = 1.7598 - 0.9609 z + 0.0013 z - 0.0188 z^{-3}. \quad (29)$$

Of course, in both cases (28) and (19) the same structure of prediction polynomial

$$D(z^{-1}, \hat{\theta}) = \hat{\theta}_1 + \hat{\theta}_2 z^{-1} + \hat{\theta}_3 z^{-2} + \hat{\theta}_4 z^{-3}$$

has been adopted.

The comparison of the related traces of Figs. 5 and 6, makes evident advantages of the use of prediction polynomial adaptation. Unlike the structure of Fig. 1, in the presence of adaptation mechanism (Fig.2), the torque disturbance is completely rejected in the steady-state.
The disturbance in Fig. 4 is periodical and, according to the principle of absorption, the adequate prediction polynomial is

\[ D(z^{-1}) = z^{-13} \]  

(30)

where \( n \) denotes the number of sampling period within the disturbance period \( \tau \) (\( \tau = nT \)). In the example under consideration, \( n \) is not an integer but \( n = 13.6591 \). The system responses \( \omega_1(t) \) and \( \omega_2(t) \) are obtained for \( n = 14 \) and \( n = 13 \), respectively. In both cases the disturbance is significantly suppressed but still affects the steady-state value of the motor speed.

Let us suppose an ideal case with the load disturbance \( T_L(t) = 1.5\sin(44.879895\tau) \cdot h(t - 0.25) \) Nm and sampling period \( T = 10 \) ms, where \( n \) is the integer

\[ n = \frac{2\pi}{T \cdot 44.879895} = 14 \]

for which the adequate prediction polynomial becomes

\[ D(z^{-1}) = z^{-13} \]

The disturbance response in Fig. 8 demonstrates the efficiency of the controlling structure of Fig. 2 in disturbance absorption in ideal cases; now, effects of disturbance on the steady-state value of motor speed is entirely rejected.

![Absorption of periodic disturbance with the absorption polynomial](image)

**Fig. 8.** Absorption of periodic disturbance with the absorption polynomial \( D(z^{-1}) = z^{-13} \)

### V. Concluding Remarks

One of the main tasks of control system is to track the reference signal, in the presence of external disturbances, without steady-state error. In most practical applications, some apriori information about the class of disturbances is available. In such cases, the extraction of entire disturbance is possible by using the principle of absorption and different controlling structures of the system. The IMPACT structure, primarily developed by YaZ. Tsypkin, is especially useful for the implementation of absorption principle because in its application there are no opposite requirements between the system stability and inclusion of the internal model of expected disturbance into the control portion of the structure. Moreover, by using the on-line algorithm of disturbance identification, it is possible to include the disturbance internal model inside the IMPACT structure even in the case of a completely unknown immeasurable arbitrary disturbance.

In this paper, the IMPACT structure of the speed controlled electrical drive with asynchronous motor has been developed. We explained how problems concerning the drive design can be advantageously solved in the presence of load torque disturbance for which there is no any apriori information. It has been shown that even in this case the rejection of disturbance is possible by the on-line identification of disturbance and adaptation of disturbance internal model. The results of simulation runs exposed exceptional possibilities of the novel approach to the use of absorption principle, proposed in this paper. To emphasize the efficiency of IMPACT structure with adaptation mechanism, the relatively fast load torque disturbance and different kinds of prediction polynomial were applied ant it was shown that the best results are achieved by using the proposed procedure of adaptation of prediction polynomial coefficients. Nevertheless, results of this paper are opening new questions about the proper choice of prediction polynomial structure, since different prediction polynomials enable the entire extraction of disturbance.

In addition, after the inclusion of adaptation mechanism into the controlling structure, the system becomes nonlinear and it may cause the stability problem particularly in the presence of large reference signal and/or disturbances.

### References


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