Repetitive control systems based on IMPACT structure

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In many engineering applications there are systems with periodical disturbances as typical characteristics: military radar systems, rotation machines, mechatronics systems for data reading and writing (CD drives), etc. This paper outlines the basic concepts for compensation of arbitrary immeasurable periodical external disturbances and/or for the accurate tracking of periodical reference signals without error in the steady-state. The conventional method based on the application of IMP (Internal Model Principle) is first described briefly and then the more efficient IMPACT (Internal Model Principle and Control Together) controlling structure is proposed, which completely excludes the effects of any periodical disturbance on the steady-state value of controlled variable (system output). Besides the efficient extraction of periodical disturbances, the IMPACT structure enables achieving higher degree of system robustness. The structure design is illustrated by the extraction of periodical load torque disturbance in a speed-controlled electrical drive with DC motor.

Key words: digital control, control algorithm, system robustness, system disturbance, control system, internal model principle.

The concept “Repetitive Control Systems (RCS)” is related to the special class of feedback control system with control algorithms that are able to extract the influence of any external periodical disturbance on the steady-state value of system output and/or to track the periodical reference signal without the error in the steady state [1, 2]. Such kinds of disturbances are met in many engineering applications. For example, in speed- and position-controlled electrical drives, periodical torque disturbances may appear with the frequency of motor rotation. Unbalances that appear in rotation machines also produce periodical torque disturbances on the frequency of rotation of moving parts of machine. The frequency of energy source may also produce periodical disturbances. [1).

Different methods for compensation of periodical disturbance have been developed for continuous and digital control systems [1-5]. Some of these methods were developed for compensation of sinusoidal disturbances and they can be easily extended for multiple sinusoidal disturbances and consequently for compensation of any kind of periodical disturbances. The bandwidth of almost all physical systems lie in a low-frequency band and therefore only a certain number of first harmonics of periodical signals should be considered and compensated. According to [2], there are two main approaches to compensation of periodical disturbances. In the first approach called AFC (Adaptive Feedforward Control), the sinusoidal disturbance is suppressed by applying the additional inverse sinusoidal signal at the input of control plant. In doing so, the amplitude and phase of disturbance are estimated adaptively. The second approach is based on the application of IMP (Internal Model Principle) or principle of absorption.

In this paper, the concept of conventional RCS systems based on the application of IMP is explained first. The new internal model of sinusoidal disturbance is proposed and tested. By introducing this model into the control portion of the system, the extraction of disturbance from the steady-state value of system output is achieved after a relatively short time of transient response. Particular attention is paid to the IMPACT controlling structure designed for digitally-controlled electrical drive subjected by an external periodical load torque disturbance. It is shown that the proposed IMPACT structure is very efficient in disturbance extraction and in achieving a high level of system robustness

Principle of absorption and internal model principle (IMP)

If the disturbance can be modeled by the function representing the solution of homogenous differential or difference equation of the given order, then it is possible, in a simple and obvious way, to modify the control part of the system so to eliminate the influence of external disturbance on the steady-state value of the system output. The homogenous differential or difference equation represents the internal model of disturbance and the method of control structure modification actually implies the application of the IMP (Internal Model Principle). Thus the principle become an important contribution to the synthesis of feedback control systems [6] long time after Kulebakin’s seminal works that founded the theory of selective invariance [7, 8].

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Essential differences between the principle of absorption and IMP do not exist and their application in solving regulation problems do not require rather complicated algebraic manipulations. Note that the use of absorption principle enables the rejection of deterministic disturbances or substantial suppression of stochastic disturbances from the system output in the steady-state. According to the principle of absorption, the model of disturbance should be imbedded into the control algorithm. This is performed by including the corresponding absorption filter into the control portion of the system; at the input the absorption filter is then excited by the signal of disturbance. Consider the synthesis of absorption filter or prediction polynomial for compensation of effects of disturbance $f(t)$ in digital control systems. Suppose that the disturbance is regular. It means that the sample $f(kT)$ can be determined by a final number $m_0$ of previous samples. In that case

$$f(kT) = D(z^{-1}) f((k-1)T)$$  \hspace{1cm} (1)

where $D(z^{-1})$ is the prediction polynomial of $m_{0-1}$ order. Relation (1) is called the equation of extrapolation or prediction [9]. Now, the absorption condition for a known class of disturbances may be expressed by the compensation equation

$$\Phi(z^{-1}) F(z^{-1}) = 0, \quad t = kT \geq (\deg \Phi)T$$ \hspace{1cm} (2)

where

$$\Phi(z^{-1}) = 1 - z^{-1} D(z^{-1})$$ \hspace{1cm} (3)

represents the compensation polynomial or absorption filter, and $F(z^{-1})$ is $z$-transform of disturbance. With sufficient a priori information about the disturbance, prediction polynomial $D(z^{-1})$ is simply determined by the model of disturbance in the time domain. However, in more complicated cases, it might be difficult to adopt the appropriate model of disturbance and thus the selection of corresponding absorption filter becomes more difficult. It is shown in [1] that for deterministic disturbance the absorption polynomial is obtained as

$$\Phi(z^{-1}) = F_{den}(z^{-1}) \text{ for } F(z) = \frac{F_{num}(z^{-1})}{F_{den}(z^{-1})}. \hspace{1cm} (4)$$

If the a priori information about the disturbance is insufficient, the adaptive approach may be used, enabling the estimation of the class of disturbances [10]. Moreover, in the case of stochastic disturbances, it is possible to synthesize the adequate absorption filter. For example, a low-frequency stochastic disturbance that can be simulated by double integration of white noise will be efficiently absorbed by absorption filter $\Phi(z^{-1}) = (1 - z^{-1})^2$ which, according to (4), corresponds to the absorption of linear (ramp) disturbance.

**RCS systems based upon IMP**

The RCS’s represent the special class of control systems based upon the application of absorption principle. Namely, controlling structures with periodic disturbances (RCS) may be considered as special cases of systems having the internal model of disturbance. The practical implementation of absorption principle consists in the introduction of absorption filter into the control part of the structure in order to compensate for the given class of disturbance. In the case of RCS systems, the important assumption is that the period of periodic disturbance is constant and unchanged during the time. The successful application of absorption principle depends on the quality (or accuracy) of the used model of disturbance and the way of absorption filter implementation into the structure of the control system. Generally, this implementation is the problem of structural synthesis which has not so far been solved definitely. If the internal model of disturbance is included into the direct path of controlling structure, then the problem of system stability and tracking accuracy appears [1].

![Digital control system](image1)

In the digital control system of Fig.1, $W_r(z^{-1})$ represents the pulse transfer function of the controller and $Q(z^{-1})$, $P_r(z^{-1})$ and $P_f(z^{-1})$ are polynomials in complex variable $z^{-1}$ ($Q(0) = P_f(0) = 1$, $P_r(0) \neq 0$), which describe the control plant; $r$ and $f$ denote the reference signal and disturbance, respectively. According to the principle of absorption, if one uses the controller transfer function

$$W_r(z^{-1}) = \frac{S(z^{-1})}{B_r(z^{-1}) \Phi(z^{-1})} \hspace{1cm} (5)$$

then the adequate absorption polynomial $\Phi(z^{-1})$ will ensure the zero steady-state error. When the dynamics of the reference signal and disturbance are quite different, it is necessary to determine two adequate polynomials $\Phi_r(z^{-1})$ and $\Phi_f(z^{-1})$ for the absorption of error in tracking the reference signal and for the rejection of disturbance from the steady-state value of the system output, respectively. Then the absorption polynomial $\Phi(z^{-1})$ in (5) should be

$$\Phi(z^{-1}) = \Phi_f(z^{-1}) \Phi_r(z^{-1}) \hspace{1cm} (6)$$

For periodical disturbances, the appropriate absorption polynomial must be adopted

$$\Phi(z^{-1}) = 1 - z^{-N} \hspace{1cm} (7)$$

where $N$ is the number of sampling periods within the period of disturbance. Frequently, zeros of polynomial
\( \Phi(z^{-1}) \) are located on the unit circle of the \( z \)-plane; for example, in the case of \( \Phi(z^{-1}) = 1 - z^{-1} \) and \( \Phi(z^{-1}) = (1 - z^{-1})^2 \) that correspond to constant and ramp disturbances, respectively. Therefore, polynomials \( S(z^{-1}) \) and \( \Phi(z^{-1}) \) should be mutually simple; otherwise, common zeros of these polynomials will become the poles of open-loop system transfer function and consequently it may imperil the system stability. Polynomials \( S(z^{-1}) \) and \( B(z^{-1}) \) in (5) are obtained as a solution of the Diophantine equation

\[
Q(z^{-1}) \Phi(z^{-1}) B(z^{-1}) + z^{-1} P_o(z^{-1}) S(z^{-1}) = K_{de}(z^{-1})
\]

where \( K_{de}(z^{-1}) \) represents the desired system characteristic polynomial. The outlined design procedure should enable the accurate regulation or tracking in the steady-state and a high quality of system transient response. It should be noted that, for greater values of \( N \) in (7), the solution of equation (8) might become rather difficult and the obtained polynomial \( S(z^{-1}) \) may be of very high order. While the implementation of absorption polynomial \( \Phi(z^{-1}) = 1 - z^{-N} \)

within the controller is not critical, a very high order of polynomial \( S(z^{-1}) \) may produce problems in achieving the real time operation of the controller. As aforementioned, the internal model \( 1 - z^{-N} \) includes \( N \) characteristic roots on the unit circle representing the stability boundary for discrete time control systems. These roots make system highly sensitive to unmodeled dynamics. The stability problem and extension of robust stability of RCS systems are solved by modifying the internal model by moving the characteristic roots into the unit circle. The modification is given by

\[
\Phi(z^{-1}) = 1 - q(z^{-1}) z^{-N}, \quad |q(e^{j\omega})| \leq 1, \quad \forall \omega \geq 0
\]

After modification (9), the compensation of the given class of periodical disturbances may be performed only approximately. Thus, there is the disagreement between the exact compensation of periodic disturbance and robust system performance. For the sake of brevity, the design procedure of low-frequency \( q(z^{-1}) \) will not be given here.

Instead of that, using the idea in [11], we propose the new synthesis of absorption filter for special class of periodic disturbances that may be odd or even function of time. The filter is given by

\[
\Phi(z^{-1}) = 1 + z^{-N/2}
\]

and its duration is twice shorter transient response than the conventional absorption filter (7).

Application of IMPACT structure in design of RCS systems

Fig.2 shows the special case of IMPACT controlling structure that corresponds the control plants without the transport lag (dead time). Thus the structure may be conveniently applied for digitally controlled electrical drives [1]. In that case, signal \( w_u \) models the influence of load torque disturbance on the system output \( y \) which may be shaft speed or angular position depending on the type of servomechanism.

![Figure 2. IMPACT structure of digital control system](image)

The control portion of the system of Fig.2 is given by polynomials in the complex variable \( z^{-1} \). In the IMPACT structure, the control plant \( W_u(s) \) is given by its simplified nominal discrete model

\[
W^0(z^{-1}) = \frac{z^{-1-k} P_o(z^{-1})}{Q^0(z^{-1})}
\]

developed at the low-frequency band. This model is included into the control part of the IMPACT structure as a two-input internal plant model. Signal \( \varepsilon \) estimates the effects of generalized external disturbance and uncertainness of the nominal plant model on the system output. Uncertainness of the nominal plant model can be adequately described by the multiplicative boundary of uncertainness \( \alpha(\omega) \)

\[
W(z^{-1}) = W^0(z^{-1})(1 + \delta W(z^{-1}))
\]

Then the system in Fig.2 satisfies the condition of robust stability if the nominal system is stable and if the following inequality is fulfilled

\[
\alpha(\omega) < \frac{Q^0(z^{-1}) R^0(z^{-1}) + z^{-1} P_o(z^{-1}) P_s(z^{-1})}{z^{-1} P_o(z^{-1}) P_s(z^{-1}) + Q^0(z^{-1}) D(z^{-1})} \bigg|_{z^{-1} e^{j\omega T}},
\]

\( \omega \in [0, \pi/T] \)

The robust performance is achieved by the local minor loop of the system in Fig.2. Namely, the main role of this loop is suppression of effects of the generalized disturbance on the system output. This loop comprises internal model of disturbance implicitly and two-input nominal plant model determined by polynomials \( z^{-1} P_o(z^{-1}) \) and \( Q^0(z^{-1}) \), explicitly. In the case of a control plant without the dead time, the internal model of disturbance is reduced to the prediction polynomial \( D(z^{-1}) \). The choice of this polynomial affects the robust performance of the system and effectiveness in absorption of the given class of disturbance. For example, for constant and ramp disturbances, the proper choice of prediction polynomials are \( D(z^{-1}) = 1 \) and \( D(z^{-1}) = 2 - z^{-1} \), respectively. In the case of a periodic disturbance having period \( NT \), where \( T \) is the sampling period of digital control system, the prediction polynomial is
\(D(z^{-1}) = z^{-\beta (N-1)}. \quad (12)\)

For a complex disturbance that can be represented by the superposition of two disturbances having prediction polynomials \(D_1(z^{-1})\) and \(D_2(z^{-1})\), the resulting prediction polynomial becomes

\[D(z^{-1}) = D_1(z^{-1}) + D_2(z^{-1}) - z^{-\beta} D_1(z^{-1}) D_2(z^{-1}). \quad (13)\]

According to the standard procedure of IMPACT structure synthesis, for a minimum phase control plant, polynomial \(R(z^{-1})\) should be taken on as

\[R(z^{-1}) = P_0^R(z^{-1}). \quad (14)\]

The polynomials \(P_1(z^{-1})\) and \(P_2(z^{-1})\) in the main external loop of the controlling structure in Fig.2 determine the dynamic behavior of closed-loop system and these polynomials are determined independently from the design of local inner control loop of the structure. The desired pole spectrum of the closed-loop control system may be specified by the relative damping coefficient \(\zeta\) and undamped natural frequency \(\omega_n\) of the system dominant poles. In doing so and taking into account the required zero steady-state error for step reference signal, the desired second order discrete closed-loop system transfer function becomes

\[G_{de}(z^{-1}) = \frac{1 - (z_1 + z_2) + z_1 z_2}{1 - (z_1 + z_2) z^{-1} + z_1 z_2 z^{-2}} = \frac{z^{-1} P_1(z^{-1})}{Q_0^R(z^{-1}) + z^{-1} P_2(z^{-1})}, \]

where

\[z_{1/2} = e^{j\omega_n T}, \quad s_{1/2} = -j \omega_n \pm j \omega_n \sqrt{1 - \zeta^2} \quad (15)\]

Then polynomials \(P_1(z^{-1})\) and \(P_2(z^{-1})\) are calculated in a straightforward manner from

\[P_1(z^{-1}) = (1 - (z_1 + z_2) + z_1 z_2) z^{-1}, \quad P_2(z^{-1}) = (1 - (z_1 + z_2) + z_1 z_2) z^{-1} - \quad (16)\]

It should be noted that the choice of dominant poles or \(\zeta\) and \(\omega_n\) determines the quality of system set-point transient response but it also affects the system robustness and filtering capability. The possible modifications of the structure in Fig.2 in order to minimize the contamination of the system due to measuring noise will not be considered here; the attention is focused only on the design of IMPACT structure in accordance with requirements of the RCS systems.

**Illustrative example**

The design of IMPACT structure will be illustrated by the synthesis of digitally-controlled speed servomechanism with the DC motor. The robot motor U12M4T having transfer function \(W_m(s) = K(T_m s + 1)\) with \(K = 4.38\) and \(T_m = 0.32s\) is adopted. To illustrate the capability and efficiency of the IMPACT structure in rejection of any kind of periodical disturbance, the elimination of the even trapezoidal external disturbance shown in Fig.3 is required.

The desired closed-loop system transfer function is specified by its dominant poles having \(\zeta = 1\) and \(\omega_n = 2.5\) rad/s inside the primary strip of the \(s\)-plane. With the sampling period \(T = 0.1s\) the desired closed-loop system transfer function becomes

\[G_{de}(z^{-1}) = \frac{0.04893z^{-2}}{1 - 1.5576z^{-1} + 0.60653z^{-2}}\]

**Figure 3. Trapezoidal torque disturbance**

The nominal model of the control plant is given by its discrete transfer function

\[W^0(z^{-1}) = Z \left[ \frac{1 - z^{-\beta}}{s} \frac{4.38}{0.52s + 1} \right]\]

which, for \(T = 0.1\), becomes

\[W^0(z^{-1}) = \frac{1.175524z^{-1}}{1 - 0.731616z^{-1}}\]

and thus

\[P_0^R(z^{-1}) = 1.175524 \quad Q_0^R(z^{-1}) = 1 - 0.731616z^{-1}\]

Since the control plant is without the dead time, \(R(z^{-1}) = P_0^R(z^{-1}) = 1.175524\). The other two polynomials \(P_1(z^{-1})\) and \(P_2(z^{-1})\) within the control part of the structure in Fig.2 are obtained directly from (15) and (16) as

\[P_1(z^{-1}) = 0.048929 \quad P_2(z^{-1}) = -0.825986 + 0.606531z^{-1}\]

With calculated \(R(z^{-1})\), \(P_1(z^{-1})\), and \(P_2(z^{-1})\) the desired quality of the set-point transient response is completed. The period of disturbance in Fig.2 of 2 seconds contains 20 sampling periods of \(T = 0.1s\) and thus \(N = 20\). Hence, the absorption filter that corresponds to absorption of periodical disturbance is \(D_0(z^{-1}) = 1 - z^{-1} D_0(z^{-1}) = 1 - z^{-N}\), wherefrom the corresponding prediction polynomial \(D_1(z^{-1}) = z^{-\beta (N-1)} = z^{-19}\) is obtained.
that correspond the periodical disturbance while stays
and . Then, it can be calculated that

\[ R(z^{-1}) = P^0_z(z^{-1}) = 0.265371 \]

\[ P^0_z(z^{-1}) = 0.265371 \]

\[ Q^0(z^{-1}) = 1 - 0.93941z^{-1} \]

\[ P_r(z^{-1}) = 0.002379 \]

\[ P_s(z^{-1}) = -0.963046 + 0.904837z^{-1} \]

Fig.6 shows the set-point response and illustrates the rejection of disturbance by the combined absorption polynomial \( D_1(z^{-1}) = z^{-99} \) while \( D_2(z^{-1}) = 2 - z^{-1} \) stays unchanged.

By comparing Figures 5 - 6 it can be concluded that the smaller sampling period significantly improves the effectiveness of the IMPACT structure in suppressing periodical external disturbances.

It is interesting to observe what would happen if instead of combined absorption polynomial, the absorption polynomials \( D_1(z^{-1}) \) and \( D_2(z^{-1}) \) were separately applied. Fig.7 shows the set-point response and illustrates the extraction of disturbance when absorption polynomial \( D_1(z^{-1}) = z^{-99} \) that corresponds the periodical disturbance is applied. The set-point response and rejection of periodical disturbance of Fig.3 are illustrated in Fig.8 when absorption polynomial \( D_1(z^{-1}) = 2 - z^{-1} \) corresponding ramp disturbances is employed. Comparing the traces of Figures 6 - 8, it can be concluded that the most efficient suppression of disturbance is achieved by construction of suitable combined prediction polynomial.
advantages when compared with other methods based upon the application of internal model principle. Namely, in the application of IMPACT structure it is not necessary to make compromise between the system robustness and speed of disturbance extraction. Furthermore, the IMPACT structure enables the extraction of periodical disturbance completely. Moreover, the nominal characteristic polynomial of IMPACT structure does not depend on the internal model of disturbance. Hence, in the application of the structure for the design of RCS systems, difficulties connected with solving the Diophantine equation (8) are avoided.

References


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Периодические системы управления, обоснованы на "ИМПАЦТ" структуре

Во многих инженерных применениях возможно встретиться со системами со возмущениями периодического характера: это военные радиолокационные системы, машины с вращательными частями, мехатронические системы для памяти и записи данных и т.д. Для качественного функционирования системы настоящее значение имеет эффективность осуществления нулевой ошибки стационарного состояния системы при присутствия возмущения со стороны нагрузки и заданной величины. Тему настоящей работы представляют - цифровые управляющие алгоритмы для компенсации произвольно выбранного неизмеримого периодического возмущения и/или для точного наблюдения референтной периодической траектории. Здесь описан установленный метод, обоснованный на принципе внутренней модели, и предложен новый метод, обоснованный "ИМПАЦТ" структуре. Кроме того что эффективно удаляет периодические возмущения, "ИМПАЦТ" структура обеспечивает и эффективное устранение периодических возмущений и высокий уровень живучести и устойчивости. Эффективность предложенного метода иллюстрирована на примере ускоряющего сервомеханизма со ДЦ-двигателем в роли исполнительного органа (элемента).

Ключевые слова: цифровое управление, алгоритм управления, живучесть системы, возмущение в системе, регулирующая система, принцип внутренней модели.

Les systèmes répétitifs de contrôle basés sur la structure IMPACT

On trouve les systèmes à dérangements répétitifs dans plusieurs applications chez les travaux d’ingénieurs: systèmes de radars militaires, machines aux pièces rotatives, systèmes mécatroniques pour mémoriser et lire les données, etc. Pour assurer un bon fonctionnement du système, il est essentiel de réaliser efficacement l’erreur zéro de l’état stationnaire du système en présence du dérangement de la part de charge et des paramètres donnés. L’objet de ce papier font les algorithmes digitaux de contrôle pour la compensation du dérangement arbitraire incommensurable et/ou la poursuite précise du tracé périodique référentiel. On a décrit la méthode conventionnelle basée sur le principe du modèle interne et on a proposé un modèle nouveau, basé sur la structure dite IMPACT. Outre l’élimination efficace des dérangements répétitifs, la structure IMPACT permet d’obtenir un très haut niveau de stabilité robuste. L’efficacité de la méthode proposée est illustrée par un exemple du servomécanisme de vitesse avec le moteur DC dans le rôle de l’organe exécutif.

Mots clés: commande digitale, algorithme de commande, robustesse du système, dérangement du système, système de contrôle, principe du modèle interne.