Structural Design of Digital Control System with Immeasurable Arbitrary Disturbances

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Abstract - This paper presents the design of control systems with uncertainties of plant parameters and immeasurable arbitrary external disturbances. To enable the zero steady state error in tracking of a reference signal, the concept of internal model is applied, which utilizes the principle of disturbance absorption. Tsypkin's IMPACT control structure (Internal Model Principle and Control Together) is employed in the case of minimal phase stable plant and presence of an arbitrary class of unmeasured disturbances. The adaptation mechanism is introduced into the control portion of the system in order to reject as much as possible the influence of disturbance on the controlled variable. The control structure proposed in this paper enables the efficient extraction of disturbance and reveals a higher degree of robustness with respect to uncertainties of plant parameters.

Index terms - IMPACT structure, generalized disturbance, absorption principle, adaptation algorithm.

I. INTRODUCTION

The main task in control system design is the tracking of a reference signal with sufficiently small or zero steady-state error in the presence of unknown disturbance. In many applications, the designer knows an amount of apriori information about the class of disturbances and limits of intervals changes or uncertainties of plant parameters. It is known that the knowledge about missmaches of plant parameters is necessary for the robustness analysis in the design of control systems with internal models [1], [2]. Most frequently an external disturbance may be modeled as a solution of homogenous differential or difference equation. In such case, it is possible to include the disturbance model into the control portion of the system in order to extract completely influence of disturbance on the steady-state value of system output [1]. Essentially, this design procedure is based upon the application of the IMP (Internal Model Principle).

The idea of absorption principle has been primarily presented in the seminal work of Kulebakin [3] who examined the invariance conditions and introduced the concept of so-called selective invariance in the control system theory. Johnson [4] used this principle in the servomechanism design. Davison [5] applied a similar approach of using the internal disturbance model for the rejection of disturbance. Solving the regulator problem in multivariable control systems, Francis and Wonham [6], [7] came to a conclusion that for disturbance rejection it was necessary to include the disturbance model into the control part of the system. As a matter of fact, IPM was inspired by results of works by Francis and Wonham [6], [7]. In [4] - [7], internal models of disturbance are given by state variables. For the first time, Bengtsson [8] applied IMP in the frequency domain. Furthermore, in a survey paper [9], IMP is approved as a fundamental concept in control system theory. Utilizing results of Shannon [10] and Kulebakin [3], Tsypkin [11] expressed concisely the principle of absorption for digital control systems. At the same time, he defined the generalized disturbance and examined the possibility of its extraction [2]. The absorption principle has been advantageously applied in the design of a robust Smith predictor for a integrating process with long dead time [12]. The IMP and principle of absorption are based upon the same fundamental idea of inclusion disturbance model into the system controlling structure.

In this paper, the principle of absorption is formulated in a discrete form and then it is applied to reject unmeasured disturbances in digital control systems. To enable extraction of an apriori unknown disturbance, the suitable modification of IMPACT structure is proposed. Within the control portion of the structure, the real-time adaptation mechanism is included, which estimates the disturbance and adapts the internal disturbance model. In [1], the general design algorithm of IMPACT structure is given in details. In this paper, the design procedure is related to only digital control systems with stable and minimal phase plants.

II. PRINCIPLE OF ABSORPTION

Suppose that kth sample of external disturbance \( w(t) \) may be determined by finite number \( m_0 \) of previous samples. Then, the disturbance is regular and may be described by extrapolation equation [10]

\[
w(kT) = D_m(z^{-1})w((k-1)T)
\]

(1)

where \( D_m(z^{-1}) \) is the prediction polynomial of order \( m_o -1 \). Equation (1) may be rewritten as
where \( w(z^{-1}) \) denotes the z-transform of disturbance. Relation (2) is called compensation equation and FIR filter having the pulse transfer function \((1-z^{-1}D_{\psi}(z^{-1}))\) is the absorption filter or the compensation polynomial [2], [11], [13]. Absorption filter \( \Phi_{\psi}(z^{-1}) = 1 - z^{-1}D_{\psi}(z^{-1}) \) is designed for a known class of disturbances and its impulse response becomes identically equal to zero after \( n \) sampling instants, where \( n \geq m_{\psi} \). Hence, the compensation equation (2) may be considered as the absorption condition of a given class of disturbances. The condition can be expressed as

\[
\Phi_{\psi}(z^{-1})w(z^{-1}) = 0, \quad \text{for } t = kT \geq (\deg \Phi_{\psi})T.
\]

The extrapolation polynomial \( D_{\psi}(z^{-1}) \) is determined by an apriori information about disturbance \( w(t) \) [2], [11], [13]; nevertheless, it is simply resolved from (1)

\[
\Phi_{\psi}(z^{-1}) = w_{\text{dep}}(z^{-1}); \quad \text{from } w(z^{-1}) = \frac{w_{\text{max}}(z^{-1})}{w_{\text{dep}}(z^{-1})}.
\]

In the case of a stochastic disturbance \( s(t) \), absorption filter (4) should suppress as much as possible disturbances of disturbance on the system output. Thus, for a law frequency disturbance \( s(t) \), which can be generated by double integration of the white noise, an appropriate choice of absorption filter is \( \Phi_{\psi}(z^{-1}) = (1-z^{-1})^2 \) that corresponds to absorption of linear (ramp) disturbance [12], [13]. In [2], the absorption principle is generalized to enable the extraction of parameter disturbances. Namely, changes of system output due to differences between the real and nominal plants may be treated as a parameter disturbance \( \varphi_p \) that influences the system output. Thus the generalized system disturbance \( \varphi = \varphi_p + w \) comprises the external disturbances and uncertainties of plant parameters. Hence, the real plant may be considered as the nominal plant subjected to the generalized disturbance \( \varphi \). If the system is robustly stable, the parameter disturbance is regular and its absorption is possible, i.e.,

\[
\Phi_{\psi}(z^{-1})\varphi_p(z^{-1}) = 0, \quad \text{for } t = kT \geq (\deg \Phi_{\psi})T
\]

with

\[
\Phi_{\psi}(z^{-1}) = 1 - z^{-1}D_{\psi}(z^{-1})
\]

where \( \Phi_{\psi}(z^{-1}) \) and \( D_{\psi}(z^{-1}) \) denote the compensation and prediction polynomials of \( \varphi_p \), respectively.

According to (3) and (5), the absorption condition of generalized disturbance \( \varphi = \varphi_p + w \) becomes

\[
\Phi_{\psi}(z^{-1})\Phi_{\varphi}(z^{-1})\varphi(z) = 0, \quad t = kT \geq (\deg \Phi_{\psi} + \deg \Phi_{\varphi})T
\]

or

\[
\Phi(z^{-1})\varphi(z^{-1}) = 0, \quad \text{for } t = kT \geq (\deg \Phi)T
\]

where \( \Phi(z^{-1}) \) denotes discrete absorption filter of generalized disturbance.

The design of absorption filter greatly depends upon the amount of apriori information about the disturbance. If the class of disturbance is known in advance (constant, slow varying, ramp, sinusoidal with known radial frequency, etc), the absorption filter can be designed exactly; otherwise, the design of absorption filter becomes more difficult. If the class of disturbance is unknown, the disturbance is to be predicted by evaluating of disturbance signal in each sampling instant. Namely, the coefficient vector \( \tilde{\theta} \) in prediction polynomial \( D(z^{-1}, \tilde{\theta}) \) is to be fitted by the online identification mechanism based on estimated disturbance [11]. Consequently, the fitting of prediction polynomial coefficients requires the on-line estimation of disturbance. In the case of IMPACT control structure, the information about the generalized disturbance is obtained by including the nominal plant model into the control portion of the system.

In a general case, let \( \epsilon(kT) \) be an estimated value of disturbance in sampling instant \( kT \). Then, analogously to (1), the following prediction relation is valid

\[
\epsilon(kT) = D(z^{-1}, \tilde{\theta})\epsilon((k-1)T).
\]

Relation (9) may be rewritten as vector equation

\[
\epsilon(kT) = \tilde{\theta}^T \psi_\epsilon(kT)
\]

where

\[
\psi_\epsilon(kT) = \begin{bmatrix} \epsilon((k-1)T), \epsilon((k-2)T), \ldots, \epsilon((k-m_\psi)T) \end{bmatrix}^T.
\]

The recurrence algorithm for estimation of coefficient vector \( \theta \) is given as

\[
\tilde{\theta}(kT) = \tilde{\theta}((k-1)T) + \Delta \tilde{\theta}(kT),
\]

\[
\Delta \tilde{\theta}(kT) = \Gamma_\epsilon(kT)\epsilon(kT)\psi_\epsilon(kT)
\]

where \( \Gamma_\epsilon(kT) \) is the gain matrix, which can be calculated in different ways [11]; the simplest one is based on the Kaczmarz algorithm

\[
\Gamma_\epsilon(kT) = \begin{bmatrix} \psi_\epsilon(kT)\psi_\epsilon(kT) \end{bmatrix}^{-1} 1
\]

where \( I \) denotes the unit matrix. In the presence of stochastic disturbances, it is convenient to estimate \( \tilde{\theta} \) by using the least square method [11].

### III. IMPACT STRUCTURE

Fig.1 shows the IMPACT control structure proposed by Ya.Z. Tsypkin [2], [11] - [14]. Actually, the structure in Fig.1 represents the modified primary IMPACT structure suited for control systems with stable plants. Within the control portion of the structure (encircled with dotted lines)
two internal models are included: the nominal plant model explicitly and the disturbance model embedded into the discrete filter $A(z^{-1})/C(z^{-1})$. Due to uncertainties of plant parameters, the real plant given by $W(z^{-1})$ (Fig.1) may be presented by
\[ W(z^{-1}) = W^o(z^{-1})(1 + \delta W(z^{-1})) \] (14)

![Fig.1. IMPACT control structure](image)

where
\[ W^o(z^{-1}) = \frac{z^{-1-k}P^o_y(z^{-1})}{Q^o(z^{-1})} \] (15)
represents the nominal plant model. Its perturbation is limited by the multiplicative bound of uncertainties.

\[ |\delta W(e^{-j\omega t})| \leq \alpha(\omega), \omega \in [0,\pi/T]. \] (16)

According to Fig.1 and Eqs. (14) and (15), one derives the error signal $e(z^{-1})$ as
\[ e = r - y = \frac{RQ^oC_y - z^{-1-k}P_yP^o_yA_y}{C_y(z^{-1-k}P_yP^o_y + RQ^o)}r \]
\[ - \frac{RQ^oC - z^{-1-k}P_yP^o_yA}{C(y)}(w + uW^o\delta W) \] (17)

where, for the sake of brevity, complex variable $z^{-1}$ is omitted from notations of variables.

As it has been shown [1],[13], polynomials $P_y(z^{-1})$ and $R(z^{-1})$ are obtained by solving the Diophantine equation [15]
\[ z^{-1-k}P_y(z^{-1})P^o_y(z^{-1}) + R(z^{-1})Q^o(z^{-1}) = K_\omega(z^{-1}) \] (18)

where $K_\omega(z^{-1})$ denotes the desired closed-loop system characteristic equation. Stable polynomials $C(z^{-1})$ and $C_y(z^{-1})$ in (17) are to be chosen according to the desired degree of system robustness with respect to mismatching of plant parameters. As a matter of fact, the determination of polynomials $P_y(z^{-1})$, $R(z^{-1})$, $C(z^{-1})$, and $C_y(z^{-1})$ may be observed as the solution of pole-placement problem [15]. The choice of polynomials $K_\omega(z^{-1})$, $C(z^{-1})$, and $C_y(z^{-1})$ influences the dynamic performance of closed-loop system and, on the other hand, it effects the robustness and filtering properties of the system. Since the reference signal $r(t)$ is not noise contaminated and because the feedforward compensator does not affects the system robust stability, one may assume $C_y(z^{-1}) \equiv 1$. A possible choice of characteristic polynomial $K_\omega(z^{-1})$ is proposed in [13] as
\[ K_\omega(z^{-1}) = \prod_{i=1}^{m} (1-b_i z^{-1})^{-1}, \quad 0 \leq b_i \leq 0.9 \] (19)

which corresponds to a strictly aperiodical closed-loop system step response. Smaller values of natural number $n$ and parameters $b_i$ correspond to higher speed of system response and lower degree of system robustness. Thus in tuning of $n$ and $b_i$, it is necessary to start with certain value of $n$ and smaller values of $b_i$ and then to increase $b_i$ gradually. If for allowable values of $b_i$ the criterion of robust stability is not satisfied, the value of $n$ should be increased to next integer and so on. Note that the system robustness may be improved by an appropriate choice of stable polynomial $C(z^{-1})$. However, polynomial $C(z^{-1})$ that improves the system robustness, at the same time, reduces the speed of disturbance absorption and vice versa.

The absorption conditions of external disturbance $w(t)$ and reference input $r(t)$ are derived from the absorption principle and relation (17) as
\[ RQ^oC - z^{-1-k}P_yP^o_yA = B\Phi \] (20)
\[ RQ^oC_y - z^{-1-k}P_yP^o_yA_y = B_y\Phi_y \]
\[ z^{-1-k}P_yP^o_yA + B\Phi = RQ^oC \]
\[ z^{-1-k}P_yP^o_yA_y + B_y\Phi_y = RQ^oC_y \] (21)

where, for the sake of brevity, $z^{-1}$ is omitted in notations of variables.
Solving Diophantine equations (21), one obtains polynomials $A_r(z^{-1})$ and $A(z^{-1})$, within the control part of IMPACT structure in Fig.1, and polynomials $B_r(z^{-1})$ and $B(z^{-1})$. Since reference signal $r(t)$ is known in advance, absorption polynomial $\Phi_r(z^{-1})$ may be chosen immediately [see Eqs. (3) and (4)]. However, absorption polynomial $\Phi(z^{-1})$ depends upon an apriori information about the class of external disturbance $w(t)$. The often choice is $\Phi(z^{-1}) = (1-z^{-1})^2$ that, according to (3) and (4), corresponds to the absorption of ramp disturbances. As it has been shown [1], [12], [13], the polynomial corresponding to absorption of ramp disturbances enables the efficient rejection of slow varying disturbances and even it suppresses affects of low frequency stochastic disturbances on the system output. Of course, the efficiency of disturbance rejection is improved by increasing the sampling rate [1]. Hence, polynomials $A_r(z^{-1})$ and $A(z^{-1})$ represent the implicit internal model of disturbances and they are determined by absorption polynomials $\Phi_r(z^{-1})$ and $\Phi(z^{-1})$, respectively [13]. It is to be noted that single solutions of Diophantine equations (18) and (21), which play crucial roles in the design procedure of IMPACT structure, do not exist. The solution existence is discussed in [15]. The procedure of solving (18) and (21), proposed in [15], yields polynomials $P_r(z^{-1})$, $R(z^{-1})$, $A_r(z^{-1})$, and $A(z^{-1})$ of lowest possible orders.

The IMPACT structure in Fig.1 may be conveniently transfigured into the equivalent one shown in Fig. 2 in order to extract signal $\epsilon$

$$\epsilon(z^{-1}) = w(z^{-1}) + u(z^{-1})W(z^{-1})\delta w(z^{-1})$$ (22)

from the output of one-input internal nominal plant model [14]. From (22), it is seen that signal $\epsilon$ represents an estimate of influences that external disturbance $w(t)$ and perturbations (or uncertainties) of plant parameters produce on the system output. Thus signal $\epsilon$ is identical to generalized disturbance and signal $u_\epsilon$ is the portion of control signal $u$, which compensates influences of generalized disturbance (22). On the other hand, signal $\hat{\epsilon}$ (Fig.2) estimates the portion of signal output that is to be compensated. The efficiency of compensation may be measured by signal

$$\xi(z^{-1}) = \epsilon(z^{-1}) - \hat{\epsilon}(z^{-1}).$$ (23)

If the IMPACT structure is able to reject the generalized disturbance completely, signal $\xi$ becomes zero in the steady state.

The steady-state value of estimation error $\xi$ of generalized disturbance will exist if absorption polynomial $\Phi(z^{-1})$ is not properly chosen due to an inadequate information about the disturbance. In such case, the adaptation mechanism based upon on-line minimization of $||\xi||$ may be utilized in order to improve the system ability in extraction of generalized disturbance [14].

![Fig.2. Equivalent IMPACT structure](image)

Fig.2 illustrates the incorporation of the proposed adaptation algorithm into the control portion of the equivalent IMPACT structure in order to adapt coefficients of polynomial

$$A(z^{-1}) = \sum_{i=0}^{\infty} a_i z^{-i}$$ (24)

with respect to external disturbance. Components $a_i$ of coefficient vector $a$ are considered as variable parameters.
of the adjustable internal model of disturbance. These parameters are adapted minimizing performance index

\[ J(a) = J(a_w, a_1, ..., a_m) = \frac{1}{2} \xi(kT)^2 \]  

by using the gradient method [16]. According to [16], the general recurrence formulas of adaptation are

\[ a_i(kT + T) = a_i(kT) + \gamma \cdot \Delta a_i(kT), \quad i = 0, 1, ..., m \]  

where \( \Delta a_i(kT) \) determines the change direction of parameter \( a_i \) at sampling instant \( kT \),

\[ \Delta a_i(kT) = -\frac{\partial J(a_i,kT)}{\partial a_i(kT)} = \xi(kT) \cdot \frac{\partial \hat{e}(kT)}{\partial a_i(kT)} \]  

while positive constant \( \gamma \) is the iteration interval. Since,

\[ \hat{e}(z^{-1}) = \frac{P_i(z^{-1})}{R(z^{-1})} \frac{z^{-1-i} P^*(z^{-1}) A(z^{-1})^2}{Q^*(z^{-1}) C(z^{-1})} \]  

it follows from (27)

\[ \Delta a_i(z^{-1}) = \frac{P_i(z^{-1})}{R(z^{-1})} \frac{z^{-1-i} P^*(z^{-1}) A(z^{-1})}{Q^*(z^{-1}) C(z^{-1})} \hat{e}(z^{-1}) \cdot \hat{e}(z^{-1}) \cdot \xi(z^{-1}). \]  

The proposed iterative procedure converges to the desired coefficient vector if positive constant \( \gamma \) satisfies the condition

\[ 0 < \gamma < \gamma_c \]  

where \( \gamma_c \) is the upper limit of convergence interval [16]. Usually, a value of \( \gamma \) is assumed by chance and then the convergence of iterative process is tested. To enable as fast as possible iterative process, the initial guess of coefficient vector \( a \) of polynomial \( A(z^{-1}, a) \) is assumed according to available apriori information about the disturbance.

IV. ILLUSTRATIVE EXAMPLE

Let the pulse transfer function of nominal plant is given by

\[ W^*(z^{-1}) = \frac{z^{-1-i} P^*(z^{-1})}{Q^*(z^{-1})} = z^{-4} - 0.7z^{-3} + 0.15z^{-2}. \]  

We assume the desired closed-loop system characteristic polynomial

\[ K_{de}(z^{-1}) = 1 - 0.3z^{-1}. \]  

The absorption filter

\[ \Phi(z^{-1}) = (1 - z^{-1})^3 \]  

is chosen to correspond to a linear approximation of generalized disturbance. For the absorption of disturbance from reference signal, the same absorption filter is assumed

\[ \Phi_r(z^{-1}) = (1 - z^{-1})^3 \]  

and then polynomials \( C_r(z^{-1}) = C(z^{-1}) = 1 \) are chosen.

Using the outlined synthesis procedure, the following polynomials of controlling structure are calculated

\[ P_i(z^{-1}) = -0.001408 - 0.0005623z^{-1} \]

\[ R(z^{-1}) = 1 + 0.4z^{-1} + 0.13z^{-2} + 0.031z^{-3} + 0.003748z^{-4} \]

\[ A(z^{-1}) = A_i(z^{-1}) = -1047 + 878.1z^{-1} \]

In the adaptation mechanism, the initial guess of polynomial \( A(z^{-1}) \) is assumed as

\[ A(z^{-1})_{0,w} = -1047 + 878.1 \cdot z^{-1} + 0 \cdot z^{-2} + 0 \cdot z^{-3}. \]  

In successive sampling instants the polynomial coefficients are adapted to external disturbance by using the proposed adaptation mechanism with the value of \( \gamma = 5000 \) chosen by simulation.
Fig. 4. Absorption of unknown external disturbance by the IMPACT structure with adaptation of internal model of disturbance.

Fig. 4 illustrates the efficiency of the IMPACT structure with the proposed adaptation mechanism in extraction of external disturbance about which we have a small amount of apriori information. The figure shows, as functions of discrete time \( kT \), reference signal \( r \), system output \( y \), tracking error \( e = r - y \), and disturbance \( w \) which is supposed to be unknown in advance. For comparison, in Fig. 5, the tracking errors are shown for ordinary IMPACT structure (solid trace) and for IMPACT structure with the adaptation mechanism (dotted trace). According to the measure of performance

\[
I = \sum_{k=0}^{200} |e(k)|
\]

(37)

the adaptation algorithm of internal disturbance model contributes for 37% to the efficiency of disturbance rejection.

Fig. 5. Tracking errors: \( e_1 \) in the presence of adaptation, \( e_2 \) without adaptation.

V. CONCLUSION

The structural design of digital control system for tracking of a reference trajectory in the presence of unknown external disturbances has been developed in this paper. The IMPACT controlling structure, which comprises both the internal nominal plant model and internal disturbance model within the control portion of the structure, is applied. The suitable adaptation mechanism of internal disturbance model is employed to enable the tracking even in the case when there is a very small amount of apriori information about the external disturbance.

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