

Suppression of Arbitrary Periodical Disturbances in the Design of Digitally Controlled Electrical Drives

Milić R. Stojić¹ and Milan S. Matijević²

Abstract – This paper outlines the basic concepts for compensation of arbitrary immeasurable periodical external disturbances and/or for the accurate tracking of periodical reference signal without error in the steady-state. The conventional method based upon the application of IMP (Internal Model Principle) is first described briefly and then the more efficient IMPACT (Internal Model Principle and Control Together) controlling structure is proposed, which completely rejects effects of any periodical disturbance on the steady-state value of controlled variable (system output). Besides the efficient extraction of periodical disturbances, the IMPACT structure enables achievement the higher degree of system robustness. The structure design is illustrated by the extraction of periodical load torque disturbance in a speed-controlled electrical drive with DC motor.

Keywords – Internal model principle, IMPACT controlling structure, Extraction of disturbance, System robustness.

1. INTRODUCTION

The concept “Repetitive Control Systems (RCS)” is related to the special class of feedback control system with control algorithms that are able to extract the influence of any external periodical disturbance on the steady-state value of system output and/or to track the periodical reference signal without the error in the steady state [1,2]. Such kind of disturbances are met in many engineering applications. For example, in speed- and position-controlled electrical drives, periodical torque disturbances may appear with the frequency of motor rotation. Unbalances that appear in rotation machines produce also periodical torque disturbances on the frequency of rotation of moving parts of machine. The frequency of energy source may also produce periodical disturbances. [1].

Different methods for compensation of periodical disturbance have been developed for continuous and digital control systems [1-5]. Some of these methods were developed for compensation of sinusoidal disturbances and they can be easily extended for multiple sinusoidal disturbances and consequently for compensation of any kind of periodical disturbances. The bandwidth of almost all physical systems lie in a low-frequency band and therefore only certain number of first harmonics of periodical signal should be considered and

considered and compensate. According to [2], there are two main approaches to compensation of periodical disturbances. In the first approach called AFC (Adaptive Feedforward Control), the sinusoidal disturbance is suppressed by applying the additional inverse sinusoidal signal at the input of control plant. In doing so, the amplitude and phase of disturbance are estimated adaptively. The second approach is based upon the application of IMP (Internal Model Principle) or principle of absorption.

In this paper, the concept of conventional RCS systems based on the application of IMP is explained first. The new internal model of sinusoidal disturbance is proposed and tested. By introducing this model into the control portion of the system, the extraction of disturbance from the steady-state value of system output is achieved after a relatively short time of transient response. Particular attention is paid to the IMPACT controlling structure designed for digitally-controlled electrical drive subjected by an external periodical load torque disturbance. It is shown that the proposed IMPACT structure is very efficient in disturbance extraction and in achieving a high level of system robustness

2. PRINCIPLE OF ABSORPTION AND INTERNAL MODEL PRINCIPLE (IMP)

If the disturbance can be modeled by the function representing the solution of homogenous differential or difference equation of given order, then it is possible, in a simple and obvious way, to modify the control part of the system so to eliminate the influence of external disturbance on the steady-state value of system output. The homogenous differential or difference equation represents the internal model of disturbance and the method of control structure modification actually means the application of the IMP (Internal Model Principle). Thus the principle become an important contribution to the synthesis of feedback control systems [6] long time after Kulebakin’s seminal works that founded the theory of selective invariance [7, 8].

Essential differences between the principle of absorption and IMP don’t exist and their application in solving regulation problems do not require rather complicated algebraic manipulations. Note that the use of absorption principle enables the rejection of deterministic disturbances or substantial suppression of stochastic disturbances from the system output, in the steady-state. According to the principle of absorption, the model of disturbance should be imbedded into the control algorithm. This is performed by including the corresponding absorption filter into the control portion of the system; at the input of absorption filter is then excited by the signal of disturbance.

Consider the synthesis of absorption filter or prediction polynomial for compensation of effects of disturbance $f(t)$ in

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digital control systems. Suppose that the disturbance is regular. It means that the sample $f(kT) = D(z^{-1})f((k-1)T)$ can be determined by a final number m_o of previous samples. In that case,

$$f(kT) = D(z^{-1})f((k-1)T) \quad (1)$$

where $D(z^{-1})$ is the prediction polynomial of m_o-1 order. Relation (1) is called the equation of extrapolation or prediction [9]. Now, the absorption condition for a known class of disturbances may be expressed by the compensation equation

$$\Phi(z^{-1})F(z^{-1}) = 0, \quad t = kT \geq (\deg \Phi)T \quad (2)$$

where

$$\Phi(z^{-1}) = 1 - z^{-1}D(z^{-1}) \quad (3)$$

represents the compensation polynomial or absorption filter, and $F(z^{-1})$ is z -transform of disturbance. With a sufficient apriori information about the disturbance, prediction polynomial $D(z^{-1})$ is simply determined by the model of disturbance in the time domain. However, in more complicated cases, it might be difficult to adopt the appropriate model of disturbance and thus the selection of corresponding absorption filter becomes more difficult. It is shown in [1] that for deterministic disturbance the absorption polynomial is obtained as

$$\Phi(z^{-1}) = F_{den}(z^{-1}), \quad F(z^{-1}) = \frac{F_{num}(z^{-1})}{F_{den}(z^{-1})}. \quad (4)$$

If the apriori information about the disturbance is insufficient, one may use the adaptive approach which enables the estimation of class of disturbances [10]. Moreover, in the case of stochastic disturbances, it is possible to synthesized the adequate absorption filter. For example, a low-frequency stochastic disturbance that can be simulated by double integration of white noise will be efficiently absorbed by absorption filter $\Phi(z^{-1}) = (1 - z^{-1})^2$ which, according to (4), corresponds to the absorption of linear (ramp) disturbance.

3. RCS SYSTEMS BASED UPON IMP

The RCS's represent the special class of control systems based upon the application of absorption principle. Namely, controlling structures with periodic disturbances (RCS) may be considered as special cases of systems having the internal model of disturbance. The practical implementation of absorption principle consists in the introduction of absorption filter into control part of the structure in order to compensate the given class of disturbance. In the case of RCS systems, the important assumption is that the period of periodic disturbance is constant and unchanged during the time. The successful application of absorption principle depends of the quality (or accuracy) of the used model of disturbance and of the way of absorption filter implementation into the structure of control system. Generally, this implementation is the problem of structural synthesis which is not so far solved definitely. If the internal model of disturbance is included into the direct path of controlling structure, then the problem of system stability and tracking accuracy appears [1].

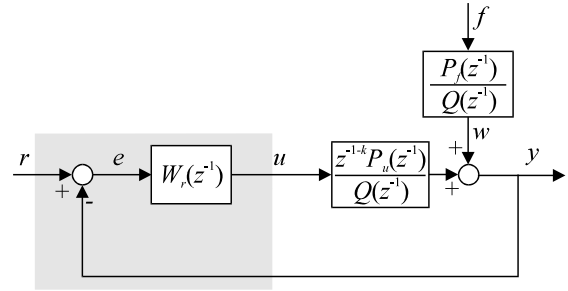


Fig.1. Digital control system

In the digital control system of Fig.1, $W_r(z^{-1})$ represents the pulse transfer function of controller and $Q(z^{-1})$, $P_u(z^{-1})$ and $P_f(z^{-1})$ are polynomials in complex variable z^{-1} ($Q(0) = P_f(0) = 1$, $P_u(0) \neq 0$), which describe the control plant; r and f denote reference signal and disturbance, respectively. According to the principle of absorption, if one uses the controller transfer function

$$W_r(z^{-1}) = \frac{S(z^{-1})}{B_1(z^{-1})\Phi(z^{-1})} \quad (5)$$

then the adequate absorption polynomial $\Phi_r(z^{-1})$ will ensure the zero steady-state error. When dynamics of reference signal and disturbance are quite different, it is necessary to determine two adequate polynomials $\Phi_r(z^{-1})$ and $\Phi_f(z^{-1})$ for the absorption of error in tracking the reference signal and for rejection of disturbance from the steady-state value of system output, respectively. Then the absorption polynomial $\Phi_r(z^{-1})$ in (5) should be

$$B(z^{-1}) = B_1(z^{-1})\Phi(z^{-1}) = B_1(z^{-1})\Phi_r(z^{-1})\Phi_f(z^{-1}). \quad (6)$$

For periodical disturbances, one must adopt the appropriate absorption polynomial

$$\Phi(z^{-1}) = 1 - z^{-N} \quad (7)$$

where N is the number of sampling periods that are contained within the period of disturbance. Frequently, zeros of polynomial $\Phi_r(z^{-1})$ are located on the unit circle of the z -plane; for example, in the case of $\Phi(z^{-1}) = 1 - z^{-1}$ and $\Phi(z^{-1}) = (1 - z^{-1})^2$ that correspond to constant and ramp disturbances, respectively. Therefore, one must take care that polynomials $S(z^{-1})$ and $\Phi_r(z^{-1})$ should be mutually simple; otherwise, common zeros of these polynomials will become the poles of open-loop system transfer function and consequently it may imperil the system stability. Polynomials $S(z^{-1})$ and $B_1(z^{-1})$ in (5) are obtained as a solution of the Diophantine equation

$$Q(z^{-1})\Phi(z^{-1})B_1(z^{-1}) + z^{-1-k}P_u(z^{-1})S(z^{-1}) = K_{de}(z^{-1}) \quad (8)$$

where $K_{de}(z^{-1})$ represents the desired system characteristic polynomial. The outlined design procedure should enable the accurate regulation or tracking in the steady-state and a high quality of system transient response. It should be noted that, for greater values of N in (7), the solution of equation (8) might become rather difficult and the obtained polynomial $S(z^{-1})$ may be of very high order. While the implementation of absorption polynomial $\Phi(z^{-1}) = 1 - z^{-N}$ within the controller is not critical, a very high order of polynomial

$$\begin{aligned} P_r(z^{-1}) &= (1 - (z_1 + z_2) + z_1 z_2) z^{-1} \\ P_y(z^{-1}) &= 1 - (z_1 + z_2) + z_1 z_2 z^{-1} \end{aligned} \quad (16)$$

It should be noted that the choice of dominant poles or ζ and ω_n determines the quality of system set-point transient response but it also affects the system robustness and filtering capability. Here we will not consider possible modifications of the structure in Fig. 2 in order to minimize the contamination of the system due to measuring noise; our attention is focused only on the design of IMPACT structure in accordance with requirements of RCS systems.

5. ILLUSTRATIVE EXAMPLE

The design of IMPACT structure will be illustrated by the synthesis of digitally-controlled speed servomechanism with the DC motor. The robot motor U12M4T having transfer function $W_{ou}(s) = K/(T_m s + 1)$ with $K=4.38$ and $T_m=0.32s$ is adopted. To illustrate the capability and efficiency of the IMPACT structure in rejection of any kind of periodical disturbance, the elimination of the even trapezoidal external disturbance shown in Fig. 3 is required.

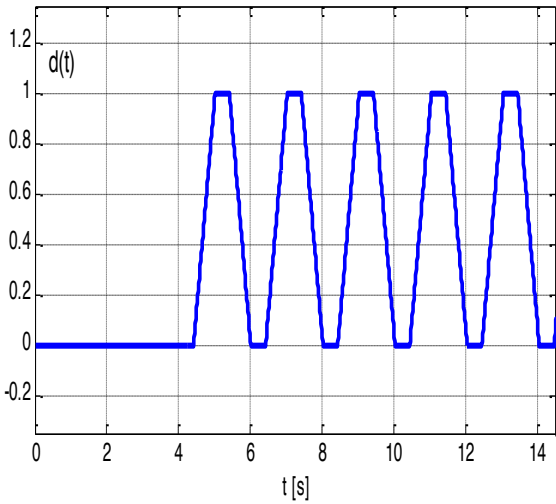


Fig. 3. Trapezoidal torque disturbance

The desired closed-loop system transfer function is specified by its dominant poles having $\zeta=1$ and $\omega_n=2.5$ rad/s inside the primary strip of the s -plane. With the sampling period $T=0.1s$ the desired closed-loop system transfer function becomes

$$G_{de}(z) = \frac{0.312898z - 0.259182}{z^2 - 1.687103z + 0.740818}$$

The nominal model of the control plant is given by its discrete transfer function

$$W^0(z^{-1}) = Z \left[\frac{1 - z^{sT}}{s} \frac{4.38}{0.32s + 1} \right]$$

which, for $T = 0.1$, becomes

$$W^0(z^{-1}) = \frac{1.175524z^{-1}}{1 - 0.731616z^{-1}}$$

and thus

$$\begin{aligned} P_u^0(z^{-1}) &= 1.175524 \\ Q^0(z^{-1}) &= 1 - 0.731616z^{-1} \end{aligned}$$

Since the control plant is without the dead time, $R(z^{-1}) = P_u^0(z^{-1}) = 1.175524$. The other two polynomials $P_r(z^{-1})$ and $P_y(z^{-1})$ within the control part of the structure in Fig. 2 are obtained directly from (15) and (16) as

$$\begin{aligned} P_r(z^{-1}) &= 0.048929 \\ P_y(z^{-1}) &= -0.825986 + 0.606531z^{-1} \end{aligned}$$

With calculated $R(z^{-1})$, $P_r(z^{-1})$, and $P_y(z^{-1})$ the desired quality of the set-point transient response is completed.

The period of disturbance in Fig. 2 of 2 seconds contains 20 sampling periods of $T = 0.1s$ and thus $N = 20$. Hence, the absorption filter that corresponds to absorption of periodical disturbance is $\Phi_1(z^{-1}) = 1 - z^{-1} D_1(z^{-1}) = 1 - z^{-N}$, wherefrom one obtains the corresponding prediction polynomial $D_1(z^{-1}) = z^{-(N-1)} = z^{-19}$.

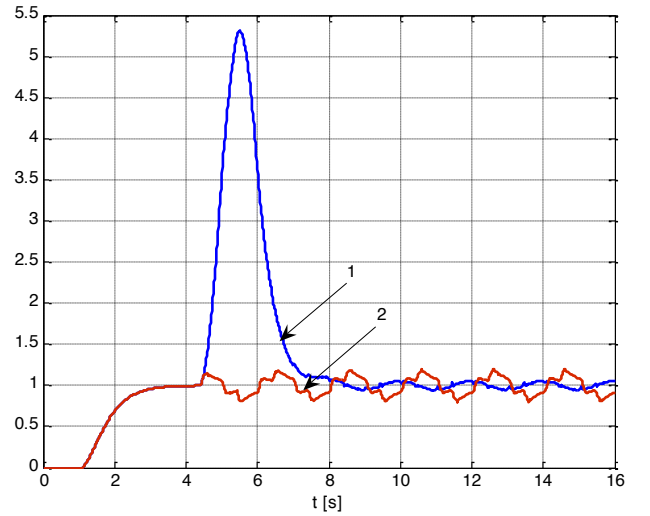


Fig. 4. Step response of the speed servomechanism and rejection of trapezoidal disturbance with prediction polynomial $D_1(z^{-1}) = z^{-19}$ (trace 1) and prediction

$$\text{polynomial } D_2(z^{-1}) = 2 - z^{-1} \text{ (trace 2)}$$

Trace 2 in Fig. 4 illustrates the absorption of disturbance in Fig. 3 when the prediction polynomial $D_2(z^{-1}) = 2 - z^{-1}$ that corresponds to the absorption of ramp (linear) disturbances is applied within the local loop of the IMPACT structure of Fig. 2.

The best result of disturbance rejection is achieved if one applies the combined absorption filter $\Phi(z^{-1}) = \Phi_1(z^{-1})\Phi_2(z^{-1})$, where $\Phi_1(z^{-1}) = 1 - z^{-20}$ and $\Phi_2(z^{-1}) = (1 - z^{-1})^2$ are absorption filters that correspond to absorption of periodical and ramp disturbances, respectively.

Thus the combined absorption polynomial $D(z^{-1})$ is derived from

$$1 - z^{-1}D(z^{-1}) = (1 - z^{-1}D_1(z^{-1}))(1 - z^{-1}D_2(z^{-1}))$$

as

$$\begin{aligned} D(z^{-1}) &= D_1(z^{-1}) + D_2(z^{-1}) - z^{-1}D_1(z^{-1})D_2(z^{-1}) \\ &= z^{-19} + 2 - z^{-1} - z^{-18}(2 - z^{-1}). \end{aligned}$$

The step set-point response and absorption of disturbance by the combined absorption polynomial are shown in Fig. 5.

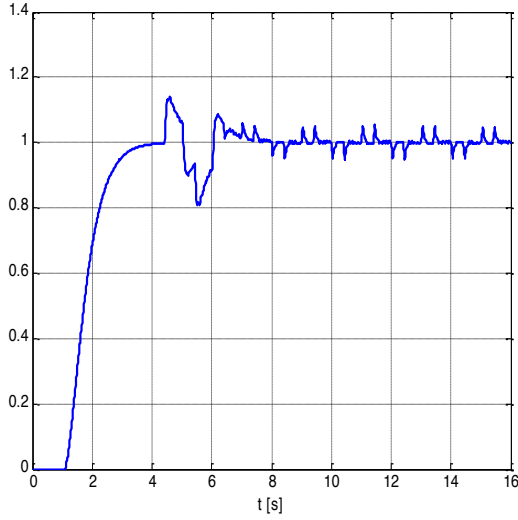


Fig. 5. Set point transient response and absorption of disturbance by the combined absorption polynomial $D(z^{-1}) = D_1(z^{-1}) + D_2(z^{-1}) - z^{-1}D_1(z^{-1})D_2(z^{-1})$ ($T = 0.1s$)

The efficiency of disturbance rejection is greatly depends upon the choice of sampling period T . Let us assume the same aperiodical step set-point response that corresponds to dominant closed-loop poles having $\zeta = 1$ and $\omega_n = 2.5$ rad/s and $T = 0.02s$. Then, one calculates

$$R(z^{-1}) = P_u^0(z^{-1}) = 0.265371$$

$$P_u^0(z^{-1}) = 0.265371$$

$$Q^0(z^{-1}) = 1 - 0.93941z^{-1}$$

$$P_r(z^{-1}) = 0.002379$$

$$P_y(z^{-1}) = -0.963046 + 0.904837z^{-1}$$

Fig. 6 shows the set-point response and illustrates the rejection of disturbance by the combined absorption polynomial $D_1(z^{-1}) + D_2(z^{-1}) - z^{-1}D_1(z^{-1})D_2(z^{-1})$ in which now $D_1(z^{-1}) = z^{-99}$ while $D_2(z^{-1}) = 2 - z^{-1}$ stayed unchanged.

By comparing Figs. 5 and 6 one can conclude that the smaller sampling period significantly improved the capability of IMPACT structure in suppressing periodical external disturbances.

It is interesting to observe what would be happened if one applies, instead of combined absorption polynomial, the

absorption polynomials $D_1(z^{-1})$ and $D_2(z^{-1})$ separately. Fig. 7 shows the set-point response and illustrates the extraction of disturbance when absorption polynomial $D_1(z^{-1}) = z^{-99}$ that corresponds to the periodical disturbance is applied. The set-point response and rejection of periodical disturbance of Fig. 3 are illustrated in Fig. 8 when absorption polynomial $D_2(z^{-1}) = 2 - z^{-1}$ corresponding to a ramp disturbances is employed. Comparing the traces of Figs. 6, 7, and 8, one can conclude that the most efficient suppression of disturbance is achieved by a construction of suitable combined prediction polynomial.

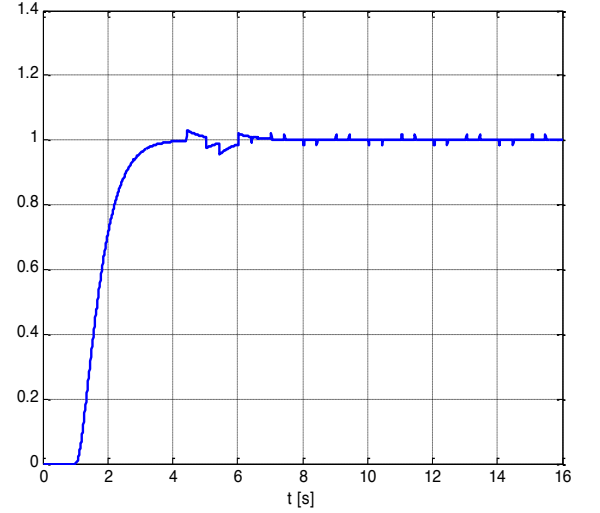


Fig. 6 Set point transient response and absorption of disturbance by the combined absorption polynomial $D(z^{-1}) = D_1(z^{-1}) + D_2(z^{-1}) - z^{-1}D_1(z^{-1})D_2(z^{-1})$ ($T = 0.02s$)

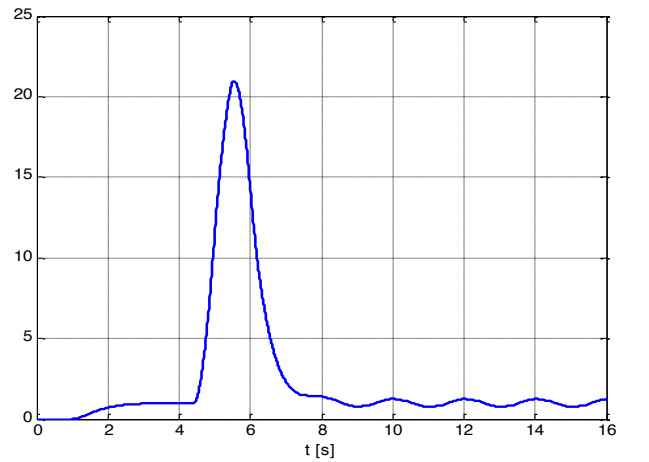


Fig. 7. Set point transient response and absorption of disturbance by absorption polynomial $D_1(z^{-1}) = z^{-99}$ ($T = 0.02s$)

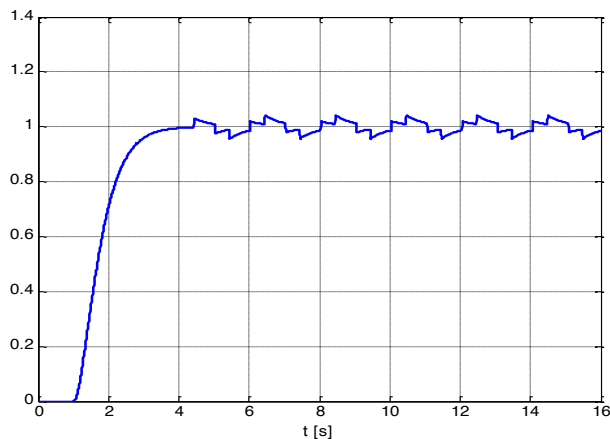


Fig. 8. Set point transient response and absorption of disturbance by absorption polynomial

$$D_2(z^{-1}) = 2 - z^{-1} \quad (T = 0.02s)$$

6. CONCLUSION

It has been shown that the application of IMPACT controlling structure in the design of RCS systems reveals advantages when compared with other methods based upon the application of internal model principle. Namely, in the application of IMPACT structure it is not necessary to make compromise between the system robustness and speed of disturbance extraction. Furthermore, the IMPACT structure enables the extraction of periodical disturbance completely. Moreover, the nominal characteristic polynomial of IMPACT structure does not depend on the internal model of disturbance. Hence, in the application of the structure for the design of RCS systems difficulties connected with the solving of the Diophantine equation (8) are avoided.

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