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COMPARATIVE CALCULATION OF CYCLOID REDUCERS EFFICIENCY BETWEEN CLASSIC AND NON-PIN WHEEL CONCEPTS

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Abstract: Minimizing power losses and defining new design concepts are vital for the further development of cycloid reducers. The aim of this paper is to compare the efficiency of a classic one-stage cycloid reducer and a relatively new non-pin wheel design. The test was performed for cycloid reducers of different sizes. Power losses on the cycloid reducer elements were taken into account, as well as the working conditions defined by different friction coefficients. The Malhotra model was used to calculate the work of the friction force in the cycloid disc bearing, on the output rollers and pins, on the rollers and pins of the central gear, and on the stationary circular segments in case of the non-pin wheel reducer. In order to calculate the forces occurring on the elements of the cycloid reducer, it was assumed that the meshing was ideal, i.e. that all the teeth of the cycloid disc were in contact and that half of them transmitted the load. The results show that the classic cycloid reducer has higher efficiency regardless of its size and the given friction coefficients.

Keywords: cycloidal reducers, efficiency, cycloidal reducer loading

1. INTRODUCTION

Cycloidal power transmissions are mechanical transmissions that are widely used in robotics, aviation, CNC machines and other branches of industry, and increasingly in electric cars [1,2]. They have good characteristics such as: compact design, low weight, high transmission ratio, high efficiency, long and reliable working life. Newer research [3,4,5] shows that due to the rotation of cycloid discs at high speed, vibrations occur on the rollers of the ring gear due to the impact of the cycloid disc on them (Figure 1). Also, due to the sliding contact between the rollers and the pins of the ring gear, as well as the inability to form an adequate oil film, greater power losses occur.

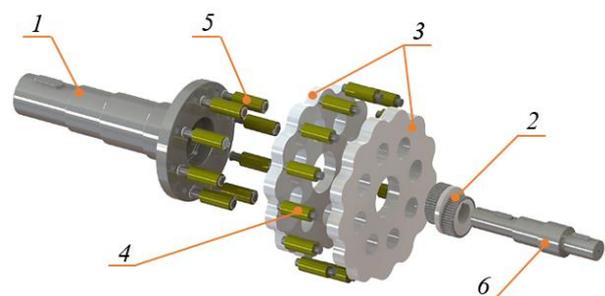


Figure 1. Single-stage cycloid reducer of the classic design (1 - input shaft, 2 - eccentric cam with needle bearings, 3 - cycloid disc, 4 - pins and rollers of the ring gear, 5 - output pins and rollers, 6 - output shaft)

Therefore, Hwang and Hsieh [6] proposed a new cycloid reducer concept where, instead of rotating rollers of the ring gear, stationary circular

segments are used that are produced flushed with the housing. A non-pin wheel concept is shown in Figure 2. In this way, the rolling friction in the contact between the teeth of the cycloid disc and the now stationary circular segments is replaced by the sliding friction. The disadvantage of this concept is that the contact stresses on the cycloid disc teeth are higher than in the case of the classic concept, and the most significant advantages are: lower shock loads, lower idle speed, lower noise and vibrations [7,8,9].

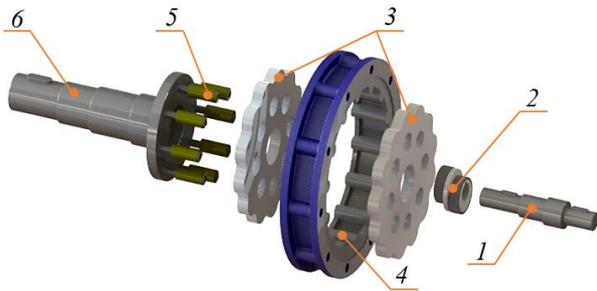


Figure 2. Single-stage cycloid reducer of non-pin wheel concept (1 - input shaft, 2 - eccentric cam with needle bearings, 3 - cycloid discs, 4 - ring gear with stationary circular segments, 5 - output pins and rollers, 6 - output shaft)

In the literature, there are many papers dealing with the efficiency of cycloid speed reducers [10,11,12]. *Kudryavtsev* [13] described the cycloid reducer in detail and presented a procedure for forces and power loss calculation. In his doctoral dissertation, *Lehmann* [14] upgraded *Kudryavtsev's* research and modified his method for force calculation. *Malhotra* [15] derived analytical expressions for calculating the total work of the friction force which served for the cycloid reducer efficiency calculation. *Davoli, Gorla,* and others [16] presented the results of the theoretical and experimental analysis of the efficiency of the new cycloid reducer concept. *Pham, Bednarczyk,* and others [17,18,19] analyzed the dependence of cycloid speed reducer efficiency on production tolerances as well as inevitable errors during production, which also occur during thermal expansions. *Blagojević* and a group of authors [20] analyzed the dependence of the efficiency on the friction coefficient in the contact between the cycloid disc and the rollers of the ring gear. *Mihailidis* [21] presented a new approach to determining efficiency, by calculating the friction at each

contact point. *Mačkić* [22] investigated the influence of geometrical parameters on the efficiency of cycloid reducers. *Matejić* and a group of authors [23] presented a procedure for the efficiency calculation of a two-stage cycloid reducer using *Autodesk INVENTOR* software. *Sensinger* [24] used optimization to increase the efficiency of cycloid reducers. *Olejarczyk* [25] analyzed the impact of installing a sliding and needle bearing on the efficiency, as well as the application of mineral and synthetic oil [26].

Based on the literature review, it can be concluded that researchers are dealing with various aspects to increase the efficiency of cycloid reducers. Therefore, this paper aims to compare the efficiency of a single-stage cycloid reducer of the classic and relatively new, non-pin wheel concept. The analysis was performed in the *MATLAB* software, in which the existing comprehensive expressions for calculating the total work of the friction force were implemented [15].

2. CYCLOID REDUCER LOADS

During the operation of the cycloid reducer, a torque (T_3) occurs on one cycloid disc as a result of driving torque ($T_1 = T_{ul} / 2$), which is shown in Figure 3. The relationship between the torques is as follows [27,28,29]:

$$T_1 - T_2 + T_3 = 0 \quad (1)$$

where: T_2 – Torque on the ring gear (Nm).

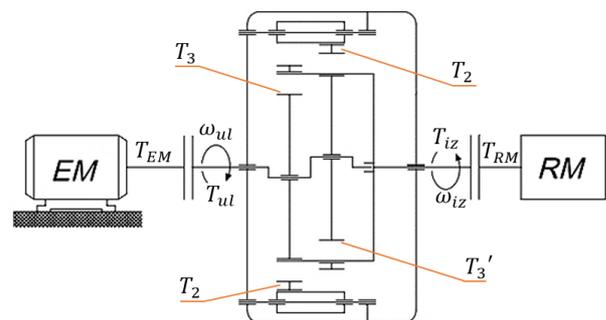


Figure 3. Kinematic diagram of cycloid speed reducer with electric motor (EM) and working machine (RM) [28]

In single-stage cycloid reducers where only one cycloid disc is used, if the power losses are ignored, the torque on the cycloid disc (T_3) is equal to the output torque (T_{iz}), that is, $T_3 = T_{iz}$.

When two cycloid discs are used in a single-stage cycloid reducer, which is the most common case, according to the recommendation from the literature [13,28], the torque on one cycloid disc (T_3) is equal to $T_3=0.55 \cdot T_{iz}$. In this way, the influence of uneven load distribution is taken into account.

The output torque can be determined from the input torque (T_{ul}) and the gear ratio (u_{CR}):

$$T_{iz} = T_{ul} \cdot |u_{CR}| \cdot \eta_{CR} \quad (2)$$

where: η_{CR} – cycloid reducer efficiency.

The torques T_1 , T_2 , and T_3 acting on the elements of the cycloid reducer produce three reactions, namely [13,14,27-34]:

- eccentric force - F_E ;
- the normal force at the current contact point between the tooth of the cycloid disc and the rollers of the ring gear - F_N ;
- normal force at the current contact point between the output rollers and holes in the cycloid disc (output force) – F_K .

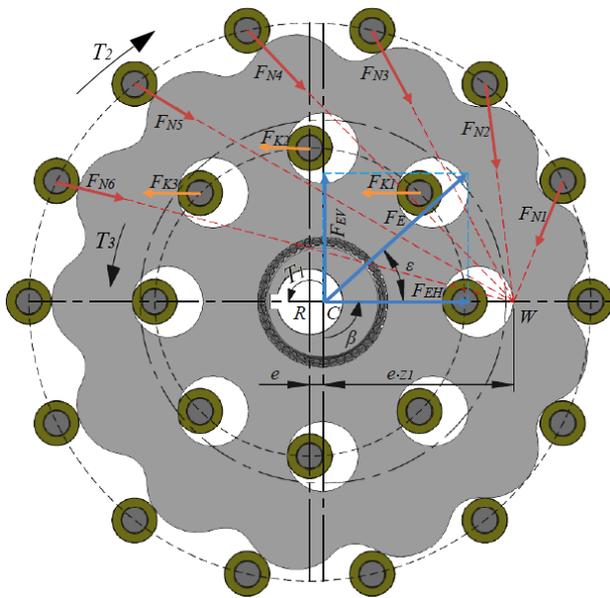


Figure 4. Forces distribution and moments acting on one of the cycloid discs in the initial position

Theoretical models from the literature are used to calculate the normal (F_N) and output force (F_K) [13,14]. However, these models do not take into account the modifications of the cycloid tooth profile as well as the elastic deformations of the elements participating in the load transfer process [27].

In the theoretical case, when there are no clearances in the cycloid reducer, both cycloid

discs are in contact with half of the rollers of the ring gear and the output rollers that participate in the load transfer process. However, in practice, this is not the case because there are clearances: due to production process tolerances, easier assembly and disassembly, and better lubrication [35]. The size of the clearance directly affects the number of rollers that are in contact with the cycloid disc, so with the increase of the clearance, the number of loaded rollers decreases [36]. Therefore, the calculated force values should be considered approximate.

The normal force on the i -th roller of the central gear is determined according to expression (3) [14,28]:

$$F_{Ni} = (c \cdot \Delta\beta) \cdot r_i \cdot \sin\psi_i \quad (3)$$

where: c – ring gear roller stiffness (N/mm); $\Delta\beta$ – cycloid disc driving angle ($^\circ$); r_i – distance between contact points of i -th ring gear roller and cycloid disc measured from cycloid disc center (mm); ψ_i – angle between normal force (F_{Ni}) direction of i -th ring gear roller and direction which connects contact point of that roller and cycloid disc with cycloid disc center ($^\circ$).

Output force on j -th roller is determined by equation [14,28]:

$$F_{Kj} = (c_k \cdot \Delta\varphi) \cdot r_{kj} \cdot \sin\psi_{kj} \quad (4)$$

where: c_k – output roller stiffness (N/mm); $\Delta\varphi$ – elementary angle movement of cycloid disc ($^\circ$); r_{kj} – distance between contact point of j -th output roller and cycloid disc measured from cycloid disc center (mm); ψ_{kj} – angle between output force (F_{Kj}) on j -th output roller and direction which connect contact point of that roller and cycloid disc with center of the cycloid disc ($^\circ$).

Eccentric force is defined as resultant of horizontal (F_{EH}) and vertical (F_{EV}) force component [14,28]:

$$F_E = \sqrt{F_{EH}^2 + F_{EV}^2} \quad (5)$$

Horizontal force component of the eccentric force is defined by equation:

$$F_{EH} = \sum_i F_{Ni} \cdot \sin x_i + \sum_j F_{Kj} \quad (6)$$

where: x_i – angle between meshing force of the i -th ring gear roller and vertical direction ($^\circ$).

Vertical eccentric force component if defined by equation:

$$F_{EV} = \frac{T_{ul}}{e} \quad (7)$$

where: e – eccentricity value (mm).

3. CYCLOID REDUCER POWER LOSSES

The procedure for determining the efficiency of the cycloid reducer is based on the calculation of the total power losses that occur due to overcoming the resistance to the movement of the rotating elements in mutual contact.

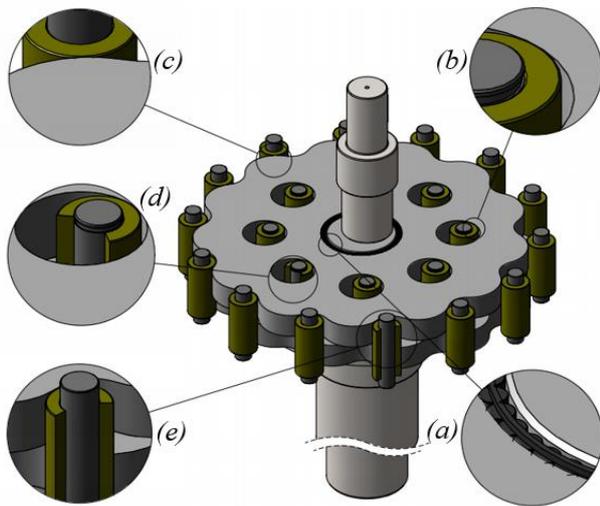


Figure 5. Contact locations that cause movement resistance in the classic cycloid reducer design concept

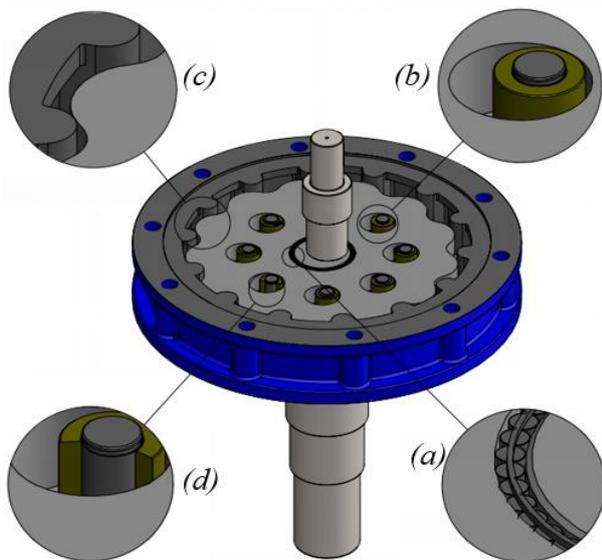


Figure 6. Contact locations that cause movement resistance in the non-pin wheel cycloid reducer design concept

Identified contacts at cycloid reducers that cause movement resistance are shown in comparison for both concepts in Table 1, while their locations are shown in Figures 5 and 6. At the same time, the power losses in the sealing elements and bearings were neglected, except for the cycloid disc needle bearing.

Table 1. Comparative contact review which describes movement resistance between rotating elements

Classic design concept	Non-pin wheel design concept
Contact of needle bearing with eccentric shaft and cycloid disc – (a) rolling friction	Contact of needle bearing with eccentric shaft and cycloid disc – (a) rolling friction
Contact of output rollers with cycloid disc – (b) (rolling friction)	Contact of output rollers with cycloid disc – (b) (rolling friction)
Contact of the ring gear rollers with cycloid disc - (c) (rolling friction)	Contact of the stationary circular segments of the ring gear with cycloid disc - (c) (sliding friction)
Contact between output rollers and pins – (d) (sliding friction)	Contact between output rollers and pins – (d) (sliding friction)
Contact between the rollers and the pins of the ring gear - (e) (sliding friction)	-

It should be emphasized that a number of contacts are common to both concepts. The main difference is that the rolling friction between the rollers of the ring gear and the cycloid disc teeth in the classical design concept is replaced by the sliding friction between the stationary circular segments of the ring gear and cycloid disc teeth. Also, with the non-pin wheel concept, the contacts between the rollers and the pins of the ring gear are lost, thus reducing the corresponding power losses.

4. MATHEMATICAL MODEL FOR DETERMINING EFFICIENCY OF CLASSIC AND NON-PIN WHEEL REDUCER CONCEPT

Determining the efficiency of the cycloid reducer [15] is based on the calculation of the

total work of the friction force (W_M) and its implementation in the equation:

$$\eta_{CR} = \frac{T_{ul} \cdot 2\pi - W_M}{T_{ul} \cdot 2\pi} \quad (8)$$

The total work of the friction force represents the integral of the total elementary work dW for a period of one revolution of the input shaft, i.e. $1/z_1$ revolution of the cycloid disc:

$$W_M = \int_0^{\frac{2\pi}{z_1}} (dW) \quad (9)$$

where: z_1 – cycloid disc tooth number.

The total elemental work dW for the classic concept includes elemental work of friction in the cycloid disc bearing (dW_1), elemental work of rolling friction between the output rollers and holes in the cycloid discs (dW_2), elemental work of rolling friction between the cycloid disc and rollers of the ring gear (dW_3), elemental work sliding friction between the output rollers and pins (dW_4) as well as the elementary work of sliding friction between the rollers and pins of the ring gear (dW_5), namely:

$$dW = dW_1 + dW_2 + dW_3 + dW_4 + dW_5 \quad (10)$$

The total elementary work dW' for the non-pin wheel concept is determined based on the modified equation:

$$dW' = dW_1 + dW_2 + dW'_3 + dW_4 \quad (11)$$

where: dW'_3 – the elementary work of sliding friction between the cycloid gear and the stationary circular segments of the ring disc.

The elementary work due to friction in the cycloid disc bearing (dW_1) can be determined based on the equation:

$$dW_1 = f_{r1} \cdot F_E(\beta) \cdot \frac{D_{SR}}{d_{kt}} \cdot z_1 \cdot d\beta \quad (12)$$

where: $f_{r1} = \mu_{r1} \cdot d_{kt} / 2$ – cycloid disc bearing friction resistance pitch (mm); μ_{r1} – friction coefficient in the cycloid disc needle bearing; $F_E(\beta)$ – current value of the eccentric force (N); β – driving angle (°); $D_{SR} = (D_{CZ} + d_{CZ}) / 2$ – cycloid disc middle diameter (mm); D_{CZ} – cycloid disc outer diameter (mm); d_{CZ} – cycloid disc inner diameter (mm); d_{kt} – rolling

body diameter (needle, cylinder, ball) in cycloid disc (mm).

The elemental work of rolling friction between the output rollers and the cycloid disc holes (dW_2) can be determined based on the equation:

$$dW_2 = f_{r2} \cdot \sum_{j=1}^q F_{Kj}(\beta) \cdot z_1 \cdot d\beta \quad (13)$$

where: $f_{r2} = \mu_{r2} \cdot D_{VK} / 2$ – output roller friction resistance pitch (mm); μ_{r2} – the coefficient of rolling friction between the output rollers and cycloid disc holes; D_{VK} – output roller diameter (mm); $F_{Kj}(\beta)$ – current output force value on the j -th output roller (N); q – the current number of output rollers that participate in the meshing and load transmission with cycloid disc. If the total number of output rollers is an even number, then it is $q = u/2$, and if it is odd, then it is $q = (u-1)/2$.

The elemental work of rolling friction between the cycloid disc and the ring gear rollers (dW_3) can be determined from the equation:

$$dW_3 = f_{r3} \cdot \sum_{i=1}^p F_{Ni}(\beta) \cdot (z_1 + 1) \cdot d\beta \quad (14)$$

where: $f_{r3} = \mu_{r3} \cdot D_0 / 2$ – friction pitch of the rolling friction of ring gear roller (mm); μ_{r3} – the coefficient of rolling friction between the rollers of the ring gear and the cycloid disc; D_0 – ring gear roller diameter (mm); $F_{Ni}(\beta)$ – the current value of the normal force on the i -th roller of the ring gear (N); p – current number of ring gear rollers participating in the load transfer process. If the total number of ring gear rollers is an even number, then it is $p = z_2/2$, and if it is odd, then it is $p = (z_2+1)/2$.

The elementary work of the sliding friction between the cycloid disc and the stationary circular segments of the ring gear (dW'_3) can be determined based on the modified equation (14), namely:

$$dW'_3 = \mu_{s3} \cdot \sum_{i=1}^p F_{Ni}(\beta) \cdot \frac{D_0}{2} (z_1 + 1) \cdot d\beta \quad (15)$$

where: μ_{s3} – the coefficient of sliding friction between the stationary circular segments and the cycloid disc.

The elementary work of sliding friction between output rollers and pins (dW_4) can be determined based on the equation:

the operation of the transmission and depend on the lubrication regime, operating temperature, the quality of the processed surfaces, the load on the rollers, and other factors.

Therefore, the impact of different values of friction coefficients on the efficiency was primarily tested.

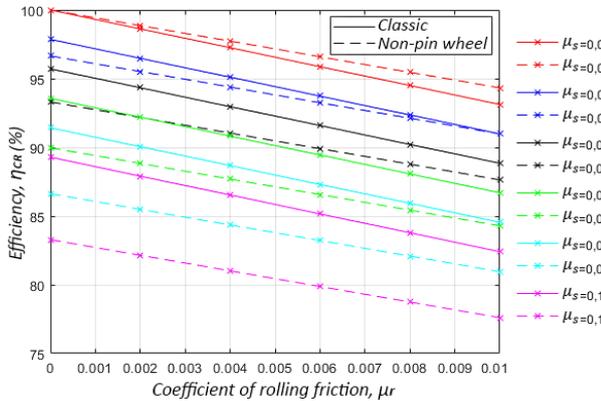


Figure 8. Dependence of the efficiency on the sliding and rolling friction coefficients

The test was performed with the simplification that all individual coefficients of sliding friction (μ_s) and rolling (μ_r) have the same values. The specific sizes of the vital elements of the cycloid reducer were used for the research and that: $D_{CZ}=40$ mm; $d_{CZ}=35$ mm; $d_{kt}=5$ mm; $z_1=13$; $z_2=14$; $D_{VK}=10,4$ mm; $d_{VK}=5,2$ mm; $D_0=14$ mm; $d_0=7$ mm; $u=8$; $P_{ul}=4$ kW.

From Figure 8, it can be concluded that the classic conception has a greater efficiency. Only in the ideal case, when there is no sliding friction ($\mu_s=0$), the non-pin wheel concept has a higher efficiency.

Table 2 gives the recommended values of sliding and rolling friction coefficients taken from the literature, while Table 3 gives their adopted values.

Table 2. Recommended values of sliding and rolling friction coefficients

References	μ_{r1}	$\mu_{r2}=\mu_{r3}$	$\mu_{s1}=\mu_{s2}$	μ_{s3}
[13]			0,07	
[15]			0,01÷0,08	
[39]	0,005			
[40]		0,006		
[41]		0,003	0,03	
[42]				0,03÷0,05

Table 3. Adopted values of sliding and rolling friction coefficients

References	μ_{r1}	$\mu_{r2}=\mu_{r3}$	$\mu_{s1}=\mu_{s2}$	μ_{s3}
[13]	0,005	0,0045	0,05	0,04

The values of the efficiency of both conceptions of the cycloid reducer for different values of the input power with $u_{CR} = 13$ and $n_{ul} = 1500$ min⁻¹ are given in Table 4.

Table 4. Values of the efficiency for different values of input power

P_{ul} , kW	Classic	Non-pin wheel	Deviation, %
2,2	90,30	89,01	1,29
3	90,48	89,17	1,31
4	91,31	90,05	1,26
5,5	91,15	89,84	1,31
7,5	92,14	90,90	1,24
11	92,22	90,93	1,29

With the increase of the input power, the efficiency of the cycloid reducer also increases. For the classic concept, the efficiency increases from 90.30% to 92.22%, and for the non-pin wheel concept from 89.01% to 90.93%. The biggest difference between the efficiency rate values is 1.31%.

The dependence of the efficiency on the input power is shown in Figure 9.

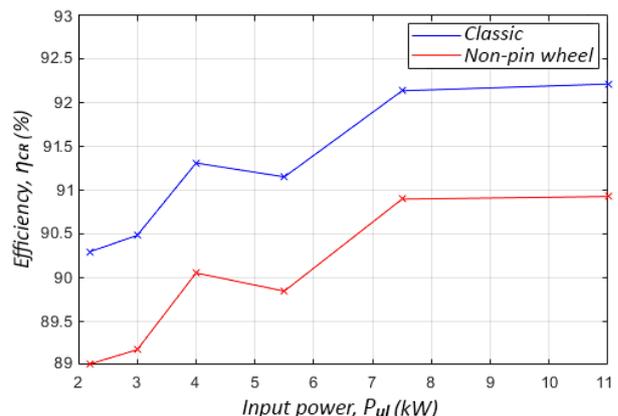


Figure 9. Dependence of the efficiency on the input power

The values of efficiency of both concepts of the cycloid reducer for different values of the transmission ratio at $P_{ul}=4$ kW and $n_{ul}=1500$ min⁻¹ are given in Table 5. With the increase of the transmission ratio, the efficiency of the cycloid

reducer decreases because the number of contacts of the rollers of the ring gear with the cycloid disc increases. For the classic concept, the efficiency decreases from 92.68% to 82.47%, and for the non-pin wheel concept from 91.60% to 80.25%.

Table 5. Values of the efficiency for different values of the transmission ratio

u_{CR}	Classic	Non-pin wheel	Deviation, %
11	92,68	91,60	1,08
13	91,31	90,05	1,26
15	89,86	88,44	1,42
17	88,42	86,82	1,60
21	85,39	83,43	1,96
25	82,47	80,25	2,22

The biggest power losses occur in the contact of the stationary circular segments with the cycloid disc ($\eta_{3'} = 4.73\% \div 4.99\%$), which causes a lower efficiency of the non-pin wheel concept. Also, significant power losses occur in the contacts of the output rollers and pins ($\eta_4 = 1.92\% \div 2.88\%$), in the contact between the rollers and pins of the ring gear ($\eta_5 = 2.96\% \div 3.12\%$), as well as in the bearing cycloid disc ($\eta_1 = 1.84\% \div 2.66\%$). Losses in the contact of the output rollers with the holes in the cycloid disc, as well as the losses in the contact of the rollers of the ring gear with the cycloid disc are significantly lower compared to the previously mentioned ones. Similar distributions of individual efficiency rates are obtained for the remaining simulations.

The dependence of the efficiency on the transmission ratio is shown in Figure 10.

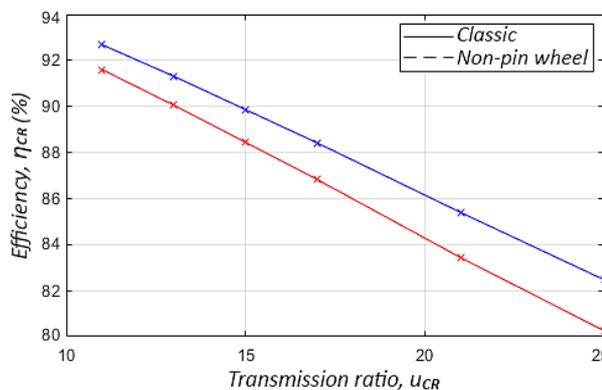


Figure 10. Dependence of the efficiency on the transmission ratio

Table 5. Values of partial losses depending on the input power

P_{ul} , kW	2,2	3	4	5,5	7,5	11
η_1 , %	2,69	2,63	2,37	2,28	2,10	1,84
η_2 , %	0,52	0,49	0,42	0,44	0,35	0,35
η_3 , %	0,55	0,56	0,54	0,56	0,53	0,55
$\eta_{3'}$, %	4,90	4,99	4,80	4,99	4,73	4,91
η_4 , %	2,88	2,72	2,36	2,45	1,92	1,96
η_5 , %	3,06	3,12	3,00	3,12	2,96	3,07

A graphical presentation of the results from Table 5 is given in Figure 11.

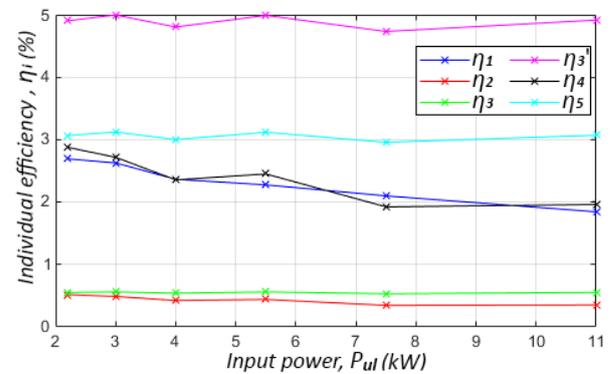


Figure 11. Dependence of partial power losses on input power

Partial power losses for different values of the transmission ratio are given in Table 6 ($P_{ul} = 4$ kW, $n_{ul} = 1500$ min⁻¹). The biggest power losses occur in the contact of stationary circular segments with the cycloid disc ($\eta_{3'} = 4.80\%$). This power loss does not depend on the input speed (n_{ul}). Also, significant power losses occur in the contacts of the output rollers and pins ($\eta_4 = 1.67\% \div 3.48\%$), in the contact between the rollers and the pins of the ring gear ($\eta_5 = 3.00\%$), as well as in the cycloid gear bearing ($\eta_1 = 2.08\% \div 2.64\%$). The graphical representation of the results from Table 6 is given in Figure 12.

Table 6. Values of partial losses depending on the transmission ratio

u_{CR}	11	13	15	17	21	25
η_1 , %	2,12	2,37	2,71	3,05	3,47	4,06
η_2 , %	0,33	0,42	0,52	0,62	0,86	1,11
η_3 , %	0,46	0,54	0,61	0,68	0,84	0,95
$\eta_{3'}$, %	4,12	4,80	5,43	6,07	7,47	8,44

$\eta_4, \%$	1,83	2,36	2,90	3,44	4,77	6,14
$\eta_5, \%$	2,57	3,00	3,39	3,79	4,67	5,28

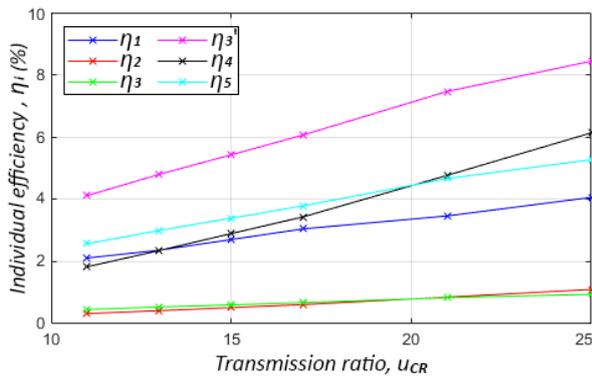


Figure 12. Dependence of partial power losses on transmission ratio

6. CONCLUSIONS

This paper presents a comparative analysis of the value of the efficiency of the classic and relatively new, non-pin wheel concept of the cycloid reducer. The test was made for different sizes of catalogue cycloid reducers.

The detailed analysis provides a good theoretical basis and helps for further improvements of the cycloid reducer because it indicates the locations of the contacts with the greatest power losses.

In the analysis, the Malhotra model is used, which takes into account the losses in the cycloid disc bearing, on the output rollers and pins, on the rollers and pins of the ring gear, that is, on the stationary circular segments when the non-pin wheel concept is in question. The results of the simulations show that the classical concept has a higher efficiency by 1.24÷1.31% when using a cycloid reducer with an input power in the interval 2.2÷11 kW, i.e. by 1.08÷2.22% when using a cycloid reducer with a transmission ratio in the interval 11÷25. The largest individual loss occurs in the contact of the stationary circular segments of the ring gear with the cycloid disc (4.12÷8.44% for the tested cycloid reducer sizes). It is precisely this loss that causes a lower efficiency of the non-pin wheel concept.

In future research, it would be interesting to find ways to reduce power losses in the contacts of stationary circular segments and

teeth of cycloid discs in the non-pin wheel concept (using adequate lubricants, better processing quality, a combination of appropriate materials...). In this way, thanks to its good features, this concept would also have greater application in modern industry.

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