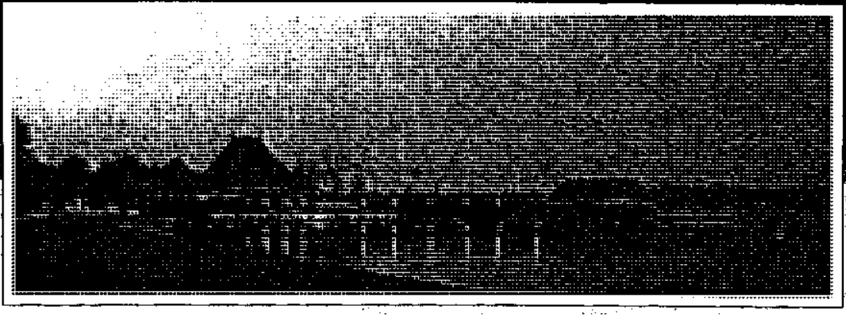
Proceedings

IConSSM 2009

2nd International Congress of Serbian Society of Mechanics

Palić (Subotica), 1-5 June 2009



Editors

Teodor Atanacković
Dragan Spasić
Srboljub Simić

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PREFACE

The present volume contains plenary lectures and papers of young authors, competing for the prize "dr Rastko Stojanović", presented at the Second International Congress of Serbian Society of Mechanics, held at Palić (Subotica) during the period 1st-5th June 2009. The Congress was organized by the Serbian Society of Mechanics. The aim of the Congress is presentation of original high level work, at the forefront of research, in various areas of Theoretical and Applied Mechanics.

During the Congress 123 papers were presented, grouped in four traditional sections: General Mechanics, Fluid Mechanics, Mechanics of Solid Bodies and Interdisciplinary and Multidisciplinary Problems. Moreover, three mini-symposia are held: (M1) MM-IX Fractional Calculus and Applications, (M2) Computational Methods in Structural Analysis and Fracture Mechanics and (M3) Computational Biomechanics. In addition, 8 invited plenary lectures were presented by the authors from France, Germany, Italy, Serbia, Slovakia and United States.

The Editors would like to thank all the authors of the papers for their active participation during the Congress, the reviewers of the papers, the members of the Scientific and Organizing Committee, the members of the Executive Committee of the Serbian Society of Mechanics, and the distinguished invited lecturers who kindly accepted the invitation to come to Congress and helped make it success.

Special thanks are also due to those organizations which supported financially this Congress: Serbian Society of Mechanics, Engineering Chamber of Serbia, Ministry of Science of Serbia, Provincial secretariat for Science and Technological Development of the Province of Vojvodina. and Municipality of Subotica.

Palić, June 2009

The Editors:
Teodor Atanacković
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2nd **International Congress of Serbian Society of Mechanics**
IConSSM 2009, Palić (Subotica), 1-5 June 2009

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AUTOMATIC ADJUSTMENT OF LOAD STEP FOR CONTACT PROBLEMS BASED ON THE PENALTY METHOD

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Abstract. In this paper methods to improve numerical simulation of contact problem by penalty method are presented. The nonlinearities of contact, which more is in large derormations inevitably, lead to the use of an incremental approach. The determination of a load step is allways very difficult. The Newton method is generally used to solve contact problems. When too many changes of contact condition occur, the algorithms of Newton Raphson family may not converge. Too many nodes coming into contact simultaneously may cause solution difficulties. Ideally, one node at a time should come into contact. The main idea of developed method is to limit the number of nodes coming during a time step by using a prediction/correction algorithm. The efficiency of the proposed algorithm is illustrated by numerical example.

1. Introduction

The effective application of finite element contact solvers need a high degree of experience since the general robustness and stability cannot be guaranteed. The analysis of contact problems is useful to improving the mechanical properties in the contact region and increasing the safety of contacted components.

The aim of this paper is to provide framework for improve contact problems with friction, based on the penalty method. The penalty formulation has the advantage that it is purely geometrically based and therefore no additional degrees of freedom must be activated or inactivated. The contact problems are load history dependent because of the irreversible nature of frictional forces. An automatic incrementation technique of the applied load has been developed and implemented in the algorithm. The size of the applied load increments, automatically chosen by the algorithm, is in general influenced both by the nature of the problem and of the discretization of the bodies involved. A load scale factor is calculated in each contact node pair where a change of contact condition will occur. The change in contact status corresponding to the node pair with the smallest load scale factor is the only change which is accomplished in a certain iteration. The uniqueness of this kind of contact problem with friction has not been mathematically proven for a general case.

Numerical example is shown to demonstrate that the presented algorithm can be successfully applied to contact problems.

2. Contact kinematics

In general, a contact can occur between: a deformable body and a rigid obstacle; between two deformable bodies or as a self-contact. In this paper a contact between two deformable bodies is considered. As the configuration of two bodies coming into the contact is not a priori known, contact represents a nonlinear problem even when the continuum behaves as a linear elastic material.

Two bodies are considered: $B^{(1)}$ and $B^{(2)}$, Fig. 1. We will denote as the contact surface $\Gamma_C^{(i)}$ the part of the body $B^{(1)}$ such that all material points where contact may occur at any time t are included.

Using a standard notation in contact mechanics we will assign to each pair of contact surfaces involved in the problem as slave and master surfaces. In particular, let $\Gamma_C^{(1)}$ be the slave surface and $\Gamma_C^{(2)}$ be the master surface. The condition which must be satisfied is that any slave particle cannot not penetrate the master surface.

Let $\bar{\mathbf{x}}$ be the projection point of the current position of the slave node $\mathbf{x}^k$ onto current position of the master surface $\Gamma_C^{(2)}$, defined as

$$\frac{\mathbf{x}^k - \bar{\mathbf{x}}(\bar{\xi}^1,\bar{\xi}^2)}{\left\|\mathbf{x}^k - \bar{\mathbf{x}}(\bar{\xi}^1,\bar{\xi}^2)\right\|}\cdot \bar{\mathbf{a}}_\alpha(\bar{\xi}^1,\bar{\xi}^2) = 0 \tag{1}$$

where $\alpha = 1,2$ and $\bar{\mathbf{a}}_\alpha(\bar{\xi}^1,\bar{\xi}^2)$ are the tangent covariant base vectors at the point $\bar{\mathbf{x}}$. These tangents are defined using following relationships:

$$\bar{\mathbf{a}}_1 = \left.\frac{\partial \bar{\mathbf{x}}}{\partial \xi^1}\right|_{\xi^1=\bar{\xi}^1,\xi^2=\bar{\xi}^2}, \quad \bar{\mathbf{a}}_2 = \left.\frac{\partial \bar{\mathbf{x}}}{\partial \xi^2}\right|_{\xi^1=\bar{\xi}^1,\xi^2=\bar{\xi}^2} \tag{2}$$

The relation (2) can be written as:

$$\bar{\mathbf{a}}_\alpha = \bar{\mathbf{x}}_{,\alpha}\left(\bar{\xi}^1,\bar{\xi}^2\right) \tag{3}$$

The definition of the projection point allows us to define the distance between any slave node and the master surface. The normal gap or the penetration g_N for slave node k is defined as the distance between current positions of this node to the master surface $\Gamma_C^{(2)}$:

$$g_N = (\mathbf{x}^k - \bar{\mathbf{x}})\cdot \bar{\mathbf{n}} \tag{4}$$

where $\bar{\mathbf{n}}$ refers to the normal to the master face $\Gamma_C^{(2)}$ at point $\bar{\mathbf{x}}$ (Fig. 1). Normal to be defined using tangent vectors at the point $\bar{\mathbf{x}}$

$$\bar{\mathbf{n}} = \frac{\bar{\mathbf{a}}_1 \times \bar{\mathbf{a}}_2}{\left\|\bar{\mathbf{a}}_1 \times \bar{\mathbf{a}}_2\right\|}. \tag{5}$$

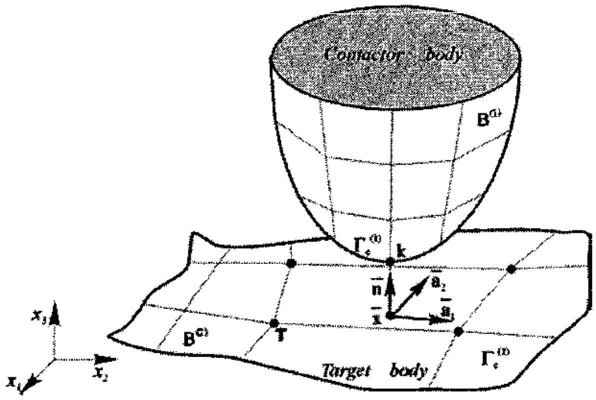


Figure 1. Geometry of the 3D node-to-segment contact element

This gap (4) gives the non-penetration conditions as follows:

$$g_N = 0 \quad \text{perfect contact;} \qquad g_N > 0 \quad \text{no contact;} \qquad g_N < 0 \quad \text{penetration} \tag{6}$$

If the analyzed problem is frictionless, function (6) completely defines the contact kinematics.

However, if friction is modeled, tangential relative displacement must be introduced. In this case the sliding path of the node $\mathbf{x}^k$ over the contact surface $\Gamma_C^{(2)}$ is described by total tangential relative displacement as

$$g_T = \int_{t_0}^{t} \left\| \dot{\mathbf{g}}_T \right\| dt = \int_{t_0}^{t} \left\| \dot{\bar{\xi}}^{\alpha} \bar{\mathbf{a}}_{\alpha} \right\| dt = \int_{t_0}^{t} \sqrt{\dot{\bar{\xi}}^{\alpha} \dot{\bar{\xi}}^{\beta} a_{\alpha\beta}}\, dt \tag{7}$$

in time interval from t_0 to t.

The time derivatives of parameter $\bar{\xi}^{\alpha}$ in equation (7) can be computed from the relation (1), [8]. In the geometrically linear case we obtain the following result

$$\frac{d}{dt}\left[(\mathbf{x}^k - \bar{\mathbf{x}}) \cdot \bar{\mathbf{a}}_{\alpha} \right] = \left[\dot{\mathbf{x}}^k - \dot{\bar{\mathbf{x}}} - \bar{\mathbf{a}}_{\beta} \dot{\bar{\xi}}^{\beta} \right] \bar{\mathbf{a}}_{\alpha} = 0 \tag{8}$$

which yields

$$\bar{a}_{\beta\alpha} \dot{\bar{\xi}}^{\beta} = \left[\dot{\mathbf{x}}^k - \dot{\bar{\mathbf{x}}} \right] \cdot \bar{\mathbf{a}}_{\alpha} = \dot{g}_{T\alpha} \tag{9}$$

where $\bar{a}_{\alpha\beta} = \bar{\mathbf{a}}_{\alpha} \cdot \bar{\mathbf{a}}_{\beta}$ is the metric tensor in point $\bar{\mathbf{x}}$ of the master surface $\Gamma_C^{(2)}$. From the equations (7) and (9) we can deduce the relative tangential velocity at the contact point

$$\dot{\mathbf{g}}_T = \dot{\bar{\xi}}^{\alpha} \bar{\mathbf{a}}_{\alpha} = \dot{g}_{T\alpha} \bar{\mathbf{a}}^{\alpha}\,. \tag{10}$$

3. Constitutive equations for contact interface

A contact stress vector $\bar{\mathbf{t}}$ with respect to the current contact interface $\Gamma_C^{(2)}$ can be split into a normal and tangential part.

$$\bar{\mathbf{t}} = \bar{\mathbf{t}}_N + \bar{\mathbf{t}}_T = \bar{t}_N \bar{\mathbf{n}} + \bar{t}_{T\alpha} \bar{\mathbf{a}}^{\alpha} \tag{11}$$

where $\bar{\mathbf{a}}^{\alpha}$ is contravariant base vector. The stress acts on both surfaces according to the action-reaction principle: $\bar{\mathbf{t}}(\bar{\xi}^1, \bar{\xi}^2) = -\mathbf{t}$ in the contact point $\bar{\mathbf{x}}$. The tangential stress $\bar{t}_{T\alpha}$ is zero in the case of frictionless contact. In the case of contact there is condition $\bar{t}_N < 0$. If there is not penetration between the bodies, then relations $g_N > 0$ and $\bar{t}_N = 0$ hold. This leads to the statements

$$g_N \geq 0, \quad \bar{t}_N \leq 0, \quad \bar{t}_N g_N = 0 \tag{12}$$

which are known as Kuhn-Tucker conditions. Using the penalty method for normal stress, constitutive equation can be formulated as

$$t_N = \varepsilon_N g_N \tag{13}$$

where ε_N is the normal penalty parameter.

In tangential direction there is difference between stick and slip. As long as no sliding between two bodies occurs, the tangential relative velocity is zero. If the velocity is zero, also the tangential relative displacement (10) is zero. This state is called stick case with the following restriction:

$$\dot{\mathbf{g}}_T = \mathbf{0} \Leftrightarrow \mathbf{g}_T = \mathbf{0} \tag{14}$$

For stick a simple linear constitutive model can be used to describe the tangential stress.

$$t_{T\alpha}^{stick} = \varepsilon_T g_{T\alpha} \tag{15}$$

where ε_T is the tangential penalty parameter.

A relative movement between two bodies occurs if the static friction resistance is overcome and the loading is large enough such that the sliding process can be kept. The tangential stress vector is restricted as follows:

$$t_{T\alpha}^{sl} = -\mu |\mathbf{t}_N| \frac{\dot{g}_{T\alpha}^{sl}}{\|\dot{\mathbf{g}}_T^{sl}\|} \tag{16}$$

where μ is friction coefficient. In the simplest form of Coulomb's law (16), μ is constant so there is no difference between static and sliding friction.

After the introduction of the stick and slip constraints, we need to introduce indicator to define whether stick or slip actually take place. Therefore an indicator function

$$f = \|\mathbf{t}_T\| - \mu |t_N| \tag{17}$$

is evaluated, which respect the Coulomb's model for frictional interface law. In the equation (17) the first term is $\|\mathbf{t}_T\| = \sqrt{t_{T\alpha}\bar{a}^{\alpha\beta}t_{T\beta}}$.

A backward Euler integration scheme and return mapping strategy are employed to integrate the friction equations (17). If a state of stick is assumed, the trial values of the tangential contact pressure vector $t_{T\alpha}$, and the indicator function f at load step *n+1* can be expressed in terms of their values at load step *n* as follows

$$t_{T\alpha\ n+1}^{trial} = t_{T\alpha\ n} + \varepsilon_T \Delta g_{T\alpha\ n+1} = t_{T\alpha\ n} + \varepsilon_T \bar{a}_{\alpha\beta} \Delta \xi_{n+1}^{\beta} \tag{18}$$

$$f_{Tn+1}^{trial} = \left\| \mathbf{t}_{Tn+1}^{trial} \right\| - \mu \left| t_{Nn+1} \right| \tag{19}$$

The return mapping is completed by

$$t_{T\alpha\ n+1} = \begin{cases} t_{T\alpha\ n+1}^{trial} & \text{if } f \le 0 \\ \mu \left| t_{Nn+1} \right| n_{T\alpha\ n+1}^{trial} & \text{if } f > 0 \end{cases} \tag{20}$$

with

$$n_{T\alpha\ n+1}^{trial} = \frac{t_{T\alpha\ n+1}^{trial}}{\left\| \mathbf{t}_{Tn+1}^{trial} \right\|} \tag{21}$$

For the both cases, the penalty method can be illustrated as a group of linear elastic springs that force the body back to the contact surface when overlapping or sliding occurs.

4. Adjustment of the load step

When too many nodes coming into contact simultaneously, it may cause solution difficulties. Ideally, one node at a time should come into contact. The basic idea is to adjust the load step according to the changes of contact conditions. The aim of the method is to limit the number of nodes coming into contact on a load step by using prediction-correction algorithm. Number of nodes N_c which get in contact for every step are defined by user. The increase or the decrease of the load step is written:

$$\Delta t_{cor} = \beta \Delta t \tag{22}$$

where β is a multiplicative factor which must be determined. The factor is calculated so that the number of nodes coming into contact on the corrected load step is limited to the given number N_c. The scalar β^k is calculated at the first Newton iteration of the time $t+\Delta t$ for a given node likely to come into contact. By considering all the nodes, β can be calculated. Then, the load step is corrected by use of β factor. The calculation is then taken again with this new load step Δt_{cor} and so the configuration at the time $t+\Delta t_{cor}$ is evaluated. Now, we explain the determination of β factor. With the goal to simplify and clearer describe determination of β factor, we suppose that reference face unchanged between times t and $t+\Delta t$ and that node remains in contact with the same face. New load step is determined so that the considered node comes exactly into contact at time $t+\Delta t_{cor}$. So, ${}^{\Delta t_{cor}}g_N^k = 0$. For each node k, likely to come into contact, the factor β^k is defined by

$$\beta^k = \frac{{}^{t}g_N^k}{{}^{t}g_N^k - {}^{t+\Delta t}g_N^k} \tag{23}$$

All the coeficients β^k are classified in an ascending order,

$$\beta^1 \le \beta^2 \le \ldots \le \beta^{N_c} \tag{24}$$

The aim of the proposed method is to limit the number of nodes coming into contact to N_c. So, the coefficient is obtained by the relation (24) is:

$$\beta = \beta^{N_c} \tag{25}$$

5. Algorithm for frictional contact

For solution a nonlinear equilibrium equation with inequality constraints (4) as a result of contact, we use a standard implicit method. In order to apply Newton's method for the solution system of equilibrium equation, a linearization of the contact contributions is necessary. In this paper, we do not state the linearization procedure for standard finite element formulation as well as the contact interface law for the normal and tangential part. It could be found in [7].

The tangent stiffness matrix for the normal contact is

$$\mathbf{K}_N = \varepsilon_N \mathbf{N}\mathbf{N}^T \tag{26}$$

The symmetric tangent stiffness matrix for stick condition is

$$\mathbf{K}_T^{stick} = \varepsilon_T \bar{a}_{\alpha\beta} \mathbf{D}^{\alpha} \mathbf{D}^{\beta T} \tag{27}$$

(20)

where

$$\mathbf{N} = \begin{Bmatrix} \bar{\mathbf{n}} \\ -H_1\bar{\mathbf{n}} \\ -H_2\bar{\mathbf{n}} \\ -H_3\bar{\mathbf{n}} \\ -H_4\bar{\mathbf{n}} \end{Bmatrix} \quad \mathbf{T}_\beta = \begin{Bmatrix} \bar{\mathbf{a}}_\beta \\ -H_1\bar{\mathbf{a}}_\beta \\ -H_2\bar{\mathbf{a}}_\beta \\ -H_3\bar{\mathbf{a}}_\beta \\ -H_4\bar{\mathbf{a}}_\beta \end{Bmatrix} \quad \mathbf{D}^{\alpha} = \bar{a}^{\alpha\beta}\mathbf{T}_\beta . \tag{28}$$

The linearization of $n_{T\alpha\ n+1}^{trial}$ gives (for details see [5])

$$\Delta\left(n_{T\alpha\ n+1}^{trial}\right) = \Delta\left(\frac{t_{T\alpha\ n+1}^{trial}}{\left\|\mathbf{t}_{Tn+1}^{trial}\right\|}\right) = \frac{1}{\left\|\mathbf{t}_{Tn+1}^{trial}\right\|}\left[\delta_{\alpha}^{\beta} - n_{T\alpha\ n+1}^{trial} n_{T\ n+1}^{trial\ \beta}\right]\Delta t_{T\beta\ n+1}^{trial} \tag{28}$$

The tangent stiffness matrix for slip condition is

$$\mathbf{K}_T^{slip} = \mu\varepsilon_N n_{T\alpha\ n+1}^{trial}\mathbf{D}^\alpha\mathbf{N}^T + \frac{\mu\varepsilon_N g_{Nn+1}}{\left\|\mathbf{t}_{Tn+1}^{trial}\right\|}\varepsilon_T\bar{a}_{\beta\gamma}\left[\delta_\alpha^\beta - n_{T\alpha\ n+1}^{trial} n_{T\ n+1}^{trial\ \beta}\right]\mathbf{D}^\alpha\mathbf{D}^{\gamma T} \quad (29)$$

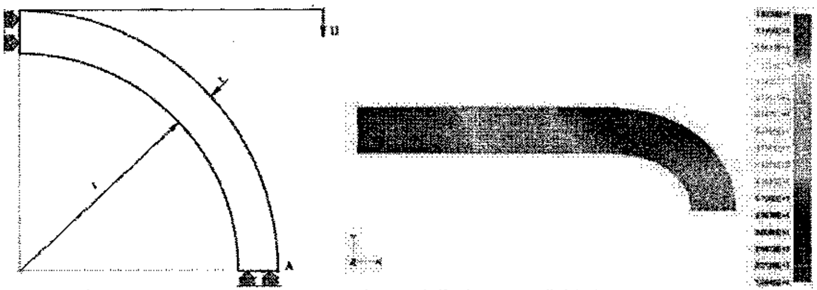
The second term, the tangent matrix is non-symmetric, due to the Coulomb's friction can be viewed as a non-associative constitutive equation.
Algorithm of the automatic adjustment of load step for frictional contact algorithm using penalty method is shown in Table 1.

Table 1. Algorithm of the automatic adjustment of load step for frictional contact algorithm using the penalty method

```
LOOP over all load step
 LOOP over all iterations
  LOOP over all contact segment k
       Determination of the penetration $g_N^k$
       IF i=1 AND I_cor=1 THEN
         Determination of $\beta^k$
         (check for contact (6))  IF $g_N \le 0$ THEN
           (the first iteration)   IF i=1 THEN
                                     set all active nodes to state stick,
                                     $\mathbf{t}_{Tn+1}$ (15), compute matrix $\mathbf{K}_T^{stick}$
                                   ELSE
                                     Compute trial state: $t_{T\alpha\ n+1}^{trial}$ (18) and $f_{Tn+1}^{trial}$ (19)
                                     IF $f_{Tn+1}^{trial} \le 0$ THEN
                                       $t_{T\alpha\ n+1} = t_{T\alpha\ n+1}^{trial}$ , compute matrix $\mathbf{K}_T^{stick}$ (27)
                                       GO TO (a)
                                     ELSE
                                       $t_{T\alpha\ n+1} = \mu|t_{Nn+1}| n_{T\alpha\ n+1}^{trial}$ , compute matrix $\mathbf{K}_T^{slip}$ (29)
                                     ENDIF
                                   ENDIF
                                ENDIF
         IF (i=1 AND I_cor=1) GO TO (b)
       ENDIF
(a)  END LOOP
(b) END LOOP
 IF I_cor=0 THEN
   Correction of load step $\Delta t_{cor} = \beta\Delta t$
   Update I_cor=1
   Automatic restart with $\Delta t = \Delta t_{cor}$
 ENDIF
END LOOP
```

6. Example

In this example banding of quarter of tube analysed. Internal tube radius is $r = 100\text{mm}$ and thickness $t = 20\text{mm}$, elastic modul $E = 400\text{MPa}$ and Poasson's ratio $\nu = 0.25$. Tube is modeled with 1313 4-node enhenced 2D elements – plain strain. Banding of the tube is conducted using rigid plate , Fig. 2a. Penalty parameter is $\varepsilon_N = 1\times10^9$. Solution is obtained by 47 steps of displacement increments equal to 1.6 mm, Fig. 2b.



a) the model b) Total displacement field, deformed configuration

Figure 2. Modle cevi

The test is realised with automatic adjustement of the step in the case of contact. The data of this adjustement is $N_c = 2$. This example ilustrates the utility of the method when the load step proposed by the user is too small. Without adjustement, 47 steps are necessary to obtain the imposed load. With presented method, one can note a very strong increase of the load step and therefore number of stpes necessary reduced to 32. The Fig. 3 illustrates this phenomenon and emphasises a non-linear evolution of load step.

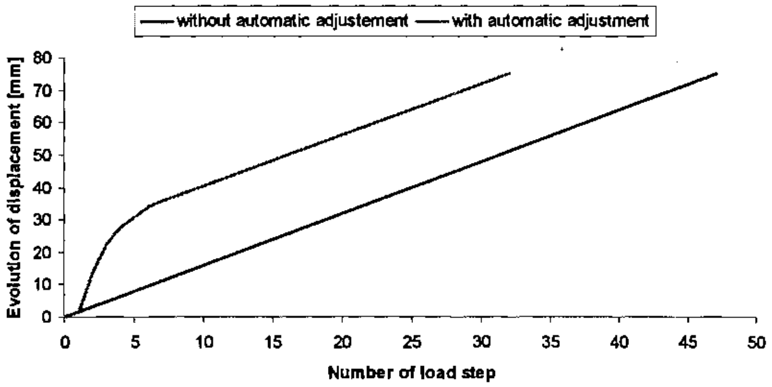


Figure 3. Evolution of load step

7. Conclusions

The automatic adjustment of load step for the numerical tretment of contact has been proposed in this paper. The load step is adjusted in order to limit the number of changes of contact status during each load step. The numerical example indicates a possibility of applying the developed method in the analysis of finite deformation problems.

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