

A FAILURE ANALYSIS OF A MASSIVE EXCAVATOR REDUCTION GEAR IN SURFACE MINING USING VIBRATION-BASED CONDITION MONITORING

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ABSTRACT

This paper presents a vibration-based failure analysis of the massive reduction gear on an excavator with a cable crowd, which had been identified as a source of high level vibrations. Vibration measurements were made in a time domain using a data acquisition (DAQ) module with an uniform time sampling period. During the measurement, the shaft rotational speed was not constant. Uniformly time-sampled vibration data were digitally resampled to an angle domain using a key phasor signal and a cubic spline interpolation algorithm. By the frequency analysis of a resampled vibration signal, damaged elements of the reduction gear were identified. A substitution of these elements, depending on the measuring points, led to vibration levels being reduced by 2.5 to 14.5 times. A cost analysis showed that the company achieved a significant cost reduction by the application of the described methods.

Keywords: reduction gear, vibration, failure analysis, signal analysis, plastic flow.

AIMS AND BACKGROUND

The present state of continuously increased global competitiveness applies significant pressure on companies for additional efforts and activities in order to ensure reduction and potential elimination of direct and indirect costs related to unscheduled downtime, unexpected breakdowns and equipment failures¹. Implementation of an appropriate maintenance strategy together with a policy of constant

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improvement of applied maintenance techniques and tools represent a necessary response to such market demands². Following the necessity for transforming the traditional, reactive, 'fail and fix' maintenance management approach to a preventive and predictive one, maintenance strategy, referred to as condition based maintenance (CBM), could be assumed as a widely accepted, appropriate and applicative solution. In its core, CBM recommends maintenance decisions based on information collected through various condition monitoring (CM) methods and techniques, which enable the identification and quantification of important degradation processes and selected parameters in technical systems³. The most widely used methods of CM are based on vibration monitoring, infrared thermography, oil analysis and wear debris analysis⁴. Further development of CBM and CM, in order to enable an appropriate response on increased demands and industry needs, will be focused on constant improvement and upgrading of measurement hardware, software and methodologies from one side, but also on introduction of advanced methods and techniques for signal processing which should support and occasionally replace traditional ones. Vibration measurement and analysis represent a first choice solution for the majority of CM procedures for rotating equipment and accordingly were used in this study to determine the condition of the investigated reduction gear for shovel lifting.

The measured vibration signal carries information about the condition of the tested reduction gear. It presents a complex combination of multiple vibration sources: shafts, gear sets, roller bearings, drive electric motors, etc. The Fast Fourier Transform (FFT), as a widely used digital signal processing (DSP) method which dominantly analysis stationary discrete signal⁵, allows each vibration component to be shown as a discrete frequency peak in the frequency spectrum. The dominant frequencies in the frequency spectrum are often related to a particular machine component or process in the system⁶, while an increase or decrease in amplitude of any vibration component may indicate failure of a specific element.

In addition, an operation load can oscillate which causes the rotational speed not to be constant during measurements. As a result, the frequency spectrum could be useless, because higher frequency components, such as gear mesh frequencies and its higher harmonics and sidebands, could be spread over several lines⁷⁻⁹. To prevent the mentioned limitation, some of the signal resampling techniques need to be applied in order to obtain constant angle domain data ($\Delta\theta$) from the uniformly time-sampled vibration data (Δt)(Refs 6, 10–13). Implementation of some of these techniques implies the introduction of additional information in the form of a signal with a key phasor which typically generates a one pulse per shaft revolution. By using the key phasor it is also possible to locate the beginning and the end of rotations and to measure their time of duration⁸.

Commonly used interpolation techniques for digital resampling of uniformly time-sampled vibration data to angle domain data ($\Delta t \rightarrow \Delta\theta$) are^{6,14}: the linear

interpolation, piecewise cubic interpolation and cubic spline interpolation. The linear interpolation, which presents a basic form of an interpolation, leads to a large amount of errors and noise amplification in the frequency spectrum, which can be avoided by using the piecewise cubic interpolation. However, this method is sensitive to inaccuracies in determining the coefficients of the interpolation polynomial. By using the cubic spline interpolation, the coefficients of an interpolation for the third order polynomial are determined in a way that the interpolation function and its first and second derivative are continuous throughout the considered interval. In this research piecewise cubic interpolation is used for angle domain digital resampling of vibration signal data.

CUBIC SPLINE INTERPOLATION

A piecewise affine function $s(x)$, which interpolates the function $y = f(x)$, consists of cubic polynomials $q_i(x)$ on the intervals $[x_{i-1}, x_i]$, $i = 1, m$, pieced together in such a way that their values and the first and second derivatives coincide at the knots (the points) $a = x_0 < x_1 < \dots < x_m = b$, so that the cubic polynomial in each subinterval depends on all data points:

$$y_j = f(x_j) = s(x_j), \quad j = 0, m. \quad (1)$$

The piecewise polynomial form of $q_i(x)$ suitable for an evaluation at many points is given by Dahlquist and Bjorck¹⁵ as:

$$q_i(x) = y_{i-1} + a_{1i}(x - x_{i-1}) + a_{2i}(x - x_{i-1})^2 + a_{3i}(x - x_{i-1})^3, \quad i = 1, m \quad (2)$$

$$a_{1i} = k_{i-1}, \quad a_{2i} = (3d_i - 2k_{i-1} - k_i)/h_i, \quad a_{3i} = (k_{i-1} + k_i - 2d_i)/h_i^2, \quad (3)$$

where $(m+1)$ unknowns: $k_0 = q'_1(x_0)$, $k_k = q'_k(x_k) = q'_{k+1}(x_k)$, $k = 1, m-1$, and $k_m = q'_m(x_m)$, are the values of the first derivatives of the spline function at the knots, $h_i = x_i - x_{i-1}$, is a step-size and $d_i = (y_i - y_{i-1})/h_i$ is a divided difference of $f(x)$.

The second derivative $q''_i(x)$ is the linear function of x . Requiring $q''_i(x)$ to be continuous at each interior knot x_k leading to $m-1$ linear equations which the $m+1$ slopes k_j must satisfy, i.e.

$$k_{k-1}(h_{k+1}) + k_k(2(h_{k+1} + h_k)) + k_{k+1}(h_k) = 3(d_k h_{k+1} + h_k d_{k+1}), \quad k = 1, m-1. \quad (4)$$

Two additional conditions needed to uniquely determine the interpolating spline can be obtained from a 'not a knot' condition for the first and last interior knot x_1 and x_{m-1} as follows¹⁵:

$$k_0(h_2) + k_1(h_2 + h_1) = 2h_2 d_1 + \frac{h_1}{h_2 + h_1}(h_2 d_1 + h_1 d_2), \quad (5)$$

$$k_{m-1}(h_{m-1} + h_m) + k_m(h_{m-1}) = 2h_{m-1} d_m + \frac{h_m}{h_{m-1} + h_m}(h_{m-1} d_m + h_m d_{m-1}). \quad (6)$$

A boundary condition for the first interior knot (5) together with equations (4) and a boundary condition for the last interior knot (6) form the three-diagonal system of linear equations which can be solved by Gaussian elimination without pivoting^{15,16} for determining slopes $k_j, j = 0, m$.

In this paper, an angle domain digital resampling procedure of the vibration data ($\Delta t \rightarrow \Delta \theta$) using cubic spline interpolation is performed in two steps. Firstly, by using the cubic spline interpolation to determine the time moments which correspond to the constant increment of the rotation angle, and after that the value of signals from vibration sensors are calculated, with the new cubic spline interpolation on these points.

EXPERIMENTAL

The reduction gear, which has been the subject of the research in this paper, is used for the shovel raising and lowering on the EKG-15 excavator. The electric excavator EKG-15 (Fig. 1) belongs to a group with cable crowd shovels. It is designed for mining and loading of overburdened rock and resources into transport facilities at surface mining, for forming mined out space. The shovel capacity is 16.5 m³ with a maximum hoist effort on the shovel of 1470 kN. The excavator working weight is 698 t, and the weight of the reduction gear of the shovel swinging mechanism is approximately 40 tons.



Fig. 1. Excavator EKG-15

A kinematics diagram of the 3-stage reduction gear of a shovel swinging mechanism is shown in Fig. 2. The two DC motors are used as a reduction gear drive. Positions for vibration measurement points (M1 to M6) are also presented in Fig. 2.

The vibrations are measured simultaneously with two BK 4391 piezoelectric charge accelerometers. These accelerometers have a frequency range from 0.1

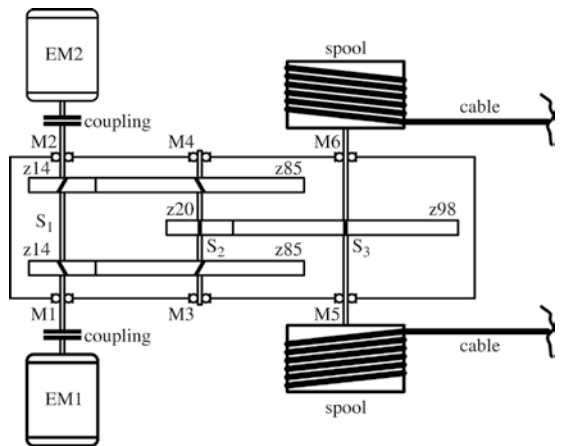


Fig. 2. Reduction gear kinematics diagram

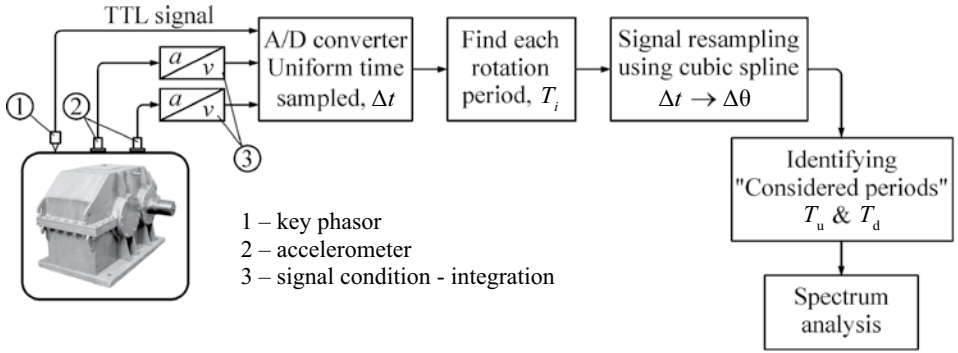


Fig. 3. Signal flow algorithm

Hz to 10 kHz and a charge sensitivity of 9.8 pC/g. Acceleration signal integration was done with a vibration signal conditioner with a 1 kHz anti-aliasing low pass filter and a velocity vibration signal was obtained. The reference shaft, where the key phasor was set, is the input shaft S_1 (see Fig. 2). The 8 analog input channels simultaneous sampling data acquisition (DAQ) module with 16-bit resolution was used for discretisation of the signal from the key phasor and two vibration sensors. The vibration signal data were measured in a vertical and axial direction with a sampling rate of 10 kHz. Duration periods of each individual rotation T_i were obtained, by using the signals from the key phasor. Uniformly time-sampled vibration data were digitally resampled to the angle domain ($\Delta t \rightarrow \Delta \theta$) by using the cubic spline algorithm. The signal flow algorithm is shown in Fig. 3.

During the investigation, shovel up and shovel down periods were considered (as it is shown in Fig. 4). During those periods, the rotational speed of the input shaft was not constant. Therefore, in each of these two periods the following values were defined:

$$\text{– for shovel up } T_u = \frac{1}{N_u} \sum_1^{n_u} T_{u_i} \quad (7)$$

$$\text{– for shovel down } T_d = \frac{1}{N_d} \sum_1^{n_d} T_{d_i} \quad (8)$$

where T_u and T_d represent the rotational speed of the input shaft mean values for considered periods (shovel up and shovel down), T_{u_i} and T_{d_i} – rotation durations for considered periods, N_u and N_d – the number of rotations for considered periods.

The frequency analysis of a vibration signal presented with constant $\Delta \theta$ will result in spectrum diagrams that usually use multiples of running speed (orders) instead of the absolute frequencies (Hz), as the frequency base⁶. In order to facilitate an interpretation of the frequency diagrams, the base frequency is multiplied with a mean value of the input shaft rotational frequency ($f_u = 1/T_u$ and $f_d = 1/T_d$). In this way the frequency base is shown in frequency values in the Hz, instead of

in orders. In such a way, some frequency components such as gear mesh frequencies and their higher harmonics have become more easily visible in the frequency spectrum.

The characteristic frequencies that can occur in the frequency spectrum of a vibration signal obtained from the reduction gear are: rotational frequencies of the shafts, gear mesh frequencies of all gear set pairs and **characteristic bearing fault frequencies**¹. Higher harmonics of these frequencies can also be presented. The rotational frequencies of the shafts for one stage reduction gear can be calculated using the following equation:

$$f_1 = \frac{n_1}{60}, \text{ Hz} \quad (9)$$

$$f_2 = \frac{n_2}{60} = f_1 \frac{z_1}{z_2}, \text{ Hz}, \quad (10)$$

where f_1 is the rotational frequency of the input shaft, f_2 – the rotational frequency of the output shaft, n_1 – the rotational speed of the input shaft, min^{-1} , n_2 – the rotational speed of the output shaft, min^{-1} , z_1 – the number of teeth on the pinion, and z_2 – the number of teeth of the gear wheel.

The following equation is used in order to calculate the characteristic gear mesh frequencies and their higher harmonics:

$$f_M = Nf_1z_1 = Nf_2z_2, \text{ Hz} \quad (11)$$

where f_M is the gear mesh frequency and N 1, 2, 3... is the fundamental frequency and its higher harmonics.

In addition, each gear set generates a series of modulations, which are presented in the frequency spectrum as sidebands that surround the fundamental gear mesh frequency. In a gear set normal condition¹, each of the sidebands is spaced by the rotational frequency of the input shaft, and sidebands always occur in pairs, one below and one above the gear mesh frequency. In addition, the amplitude of each sideband pair is identical, i.e. sidebands are symmetrically placed around the gear mesh frequency. Each reduction gear problem, such as external and internal misalignment, overloads, intensive wear, etc. destroys the symmetry of the sidebands profile¹⁷.

The roller bearing characteristic frequencies have also been considered, but since they do not appear in the frequency spectrum, we are not going to present their equations.

RESULTS AND DISCUSSION

Measurement and analysis of vibrations on the investigated reduction gear were performed through preliminary stage investigation and final stage investigation

after the realisation of necessary maintenance activities. Preliminary stage investigation results were used for a failure analysis procedure of the reduction gear in order to enable the identification and localisation of its elements which were responsible for the increased level of vibrations. The final stage investigation results were used for a comparison with preliminary stage ones, as a measurable indicator of vibration reduction and a verification of achieved results through the realisation of maintenance activities.

Preliminary stage measurement results. A typical profile of the reduction gear input shaft rotational speed changes during one shovel up/down cycle is shown in Fig. 4. The duration of the shown cycle is very close to the design cycle duration for this excavator which is 26 s. The period when the system reaches maximum rotational speeds during the shovel up and down cycle is of interest for this research, and its duration is approximately 7–8 s. The average value of the reduction gear input shaft rotational speed during the shovel up cycle has been slightly lower than the mean rotational speed during the shovel down cycle. The observed phenomenon is understandable since the shovel up process is opposed to the mass of the shovel, supporting structure and overburdened rock, while during the shovel down process, the mentioned masses accelerate the descent.

The measured values of the root mean square (rms) vibration velocity are shown in Table 1. Vibration levels during the shovel down cycle at all measurement points are from 2 to 2.5 times higher than vibration levels during the shovel up cycle. The highest measured vibration level is extremely high and it is collected during the shovel down cycle at the measurement points M5 and M6 in the axial direction. Furthermore, the high level of vibrations has also been measured in the vertical direction at the measurement points M3 and M4 during the shovel down cycle.

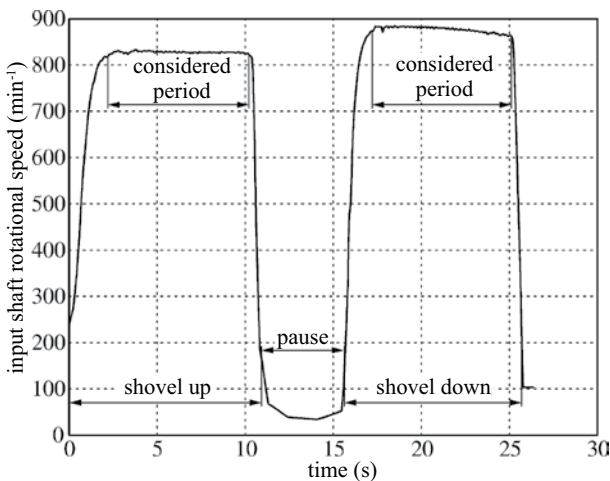


Fig. 4. Profile of the reduction gear input shaft rotational speed during the shovel up/down cycle

Table 1. Vibration levels measured on the reduction gear

Measurement point	Direction	Shovel up		Shovel down	
		\bar{n}_{ui} min ⁻¹	v_{rms} mm/s	\bar{n}_{di} min ⁻¹	v_{rms} mm/s
M1	V	821.2	14.91	870.1	31.82
	A	822.1	7.40	867.2	16.42
M2	V	827.7	16.15	876.8	29.17
	A	828.8	6.83	878.2	13.40
M3	V	821.2	16.79	870.1	35.37
	A	822.1	12.88	867.2	24.90
M4	V	827.7	19.17	876.8	34.04
	A	828.8	9.39	878.2	21.82
M5	V	819.5	24.93	868.7	33.23
	A	821.5	19.86	868.3	35.64
M6	V	828.6	23.11	874.6	29.55
	A	824.9	14.37	879.9	40.33

Note: V – vertical, A – axial, \bar{n}_{ui} , \bar{n}_{di} – rotational speed mean value of the input shaft for the considered shovel up/down period and v_{rms} – rms value of the vibration velocity

In order to determine the cause of high vibration levels, an analysis has been carried out in the frequency domain. As it has already been stated, the following characteristic frequencies can occur in the frequency spectrum: rotational frequencies of the shafts, gear mesh frequencies of all gear set pairs and the characteristic bearing fault frequencies. Since the considered periods during the shovel up/down cycles have different mean rotational speeds (see Fig. 4), it is necessary to calculate both of the mentioned characteristic frequencies for the shovel up/down periods. Based on the results given in Table 1, mean rotational speeds for both shovel up/down cycles were calculated and are shown in Table 2, while gear mesh characteristic frequencies are shown in Table 3.

Table 2. Rotational speed characteristic frequencies (Hz)

Shaft No	Shovel up	Shovel down
S1	$f_{1u} = 13.75$	$f_{1d} = 14.58$
S2	$f_{2u} = f_{1u} \frac{z_{14}}{z_{85}} = 2.26$	$f_{2d} = f_{1d} \frac{z_{14}}{z_{85}} = 2.40$
S3	$f_{3u} = f_{2u} \frac{z_{20}}{z_{98}} = 0.46$	$f_{3d} = f_{2d} \frac{z_{20}}{z_{98}} = 0.49$

Table 3. Gear mesh characteristic frequencies (Hz)

Gear mesh	Shovel up	Shovel down
f_{M1}	$f_{M1u} = f_{1u} z_{14} = 192.5$	$f_{M1d} = f_{1d} z_{14} = 204.1$
f_{M2}	$f_{M2u} = f_{2u} z_{20} = 45.2$	$f_{M2d} = f_{2d} z_{20} = 48.0$

The input shaft rotational speed is not constant in the considered periods (see Fig. 4). In this case, the application of FFT analysis to the uniformly time-sampled vibration data would not give the desired results. High frequency components in the spectrum, representing gear mesh frequencies, might be smeared so much that details in the sideband structures will be lost in the analysis^{8,9}. That is why, a uniformly time-sampled vibration data are digitally resampled to the angle domain using the cubic spline algorithm. The process of digital resampling of a vibration signal using the cubic spline interpolation is explained in details below.

Frequency spectra, which correspond to the measurement point M6 (see Fig. 2) in the vertical and axial directions in both shovel up/down cycles are shown in Fig. 5. These frequency spectra represent the oscillatory behaviour of the tested reduction gear. In order to display certain frequencies more clearly, diagrams are given in the frequency range of 0–200 Hz. Frequencies above this range have low amplitude levels and are not of interest.

As it can be seen from the frequency spectra, the gear mesh frequency of the 2nd gear set (f_{M2u} and f_{M2d}) is distinctly dominant during shovel up/down processes,

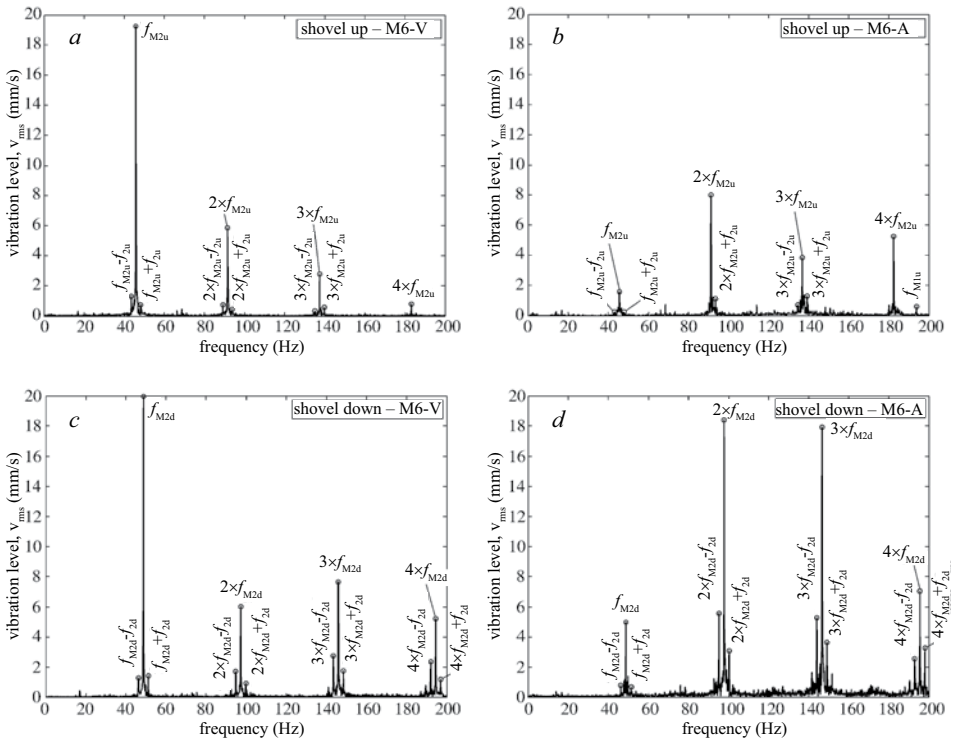


Fig. 5. Frequency spectra of the resampled vibration data on the measurement point M6 *a* – the vertical direction, shovel up, *b* – the axial direction, shovel up, *c* – the vertical direction, shovel down, and *d* – the axial direction, shovel down

with its higher harmonics (Nf_{M2u} and Nf_{M2d}). The belonging sidebands on the shaft S_2 rotational frequency, in other words $N(f_{M2u} \pm f_{2u})$ and $N(f_{M2d} \pm f_{2d})$, where $N = 1, 2, 3...$ are fundamental frequency and harmonics, are also presented. In the vertical direction, the fundamental gear mesh frequency is dominant (f_{M2u} and f_{M2d}), while the 2nd harmonics of these gear mesh frequencies is dominant in the axial direction.

The gear mesh frequency of the 1st gear set f_{M1u} can be noticed in Fig. 5b, although it is poorly expressed. This characteristic frequency can also be noticed at the measurement points M1 and M2, which are also weakly expressed.

While looking at the frequency spectra, it can be seen that the sidebands are not symmetrically placed around the gear mesh frequency. This asymmetry is due to overload and intensive wear of the 2nd gear set. High vibration levels in the axial direction suggest the presence of uneven wear of the tooth lateral side, as well as internal misalignment. It should also be noted that the vibration level during the shovel down cycle is from 2 to 2.5 times higher than the shovel up cycle vibration level. This behaviour is due to an increased clearance in the roller bearings on the S_2 and S_3 shafts, which in a combination with the intensive wear of the 2nd gear set, leads to unequal tooth contact of this gear set during shovel up/down cycles.

The above frequency spectrum analysis results helped to closely locate the cause of high vibration levels. It is without doubt the 2nd gear set and bearings on the S_2 and S_3 shafts.

Realised maintenance activities and achieved results. The investigation results obtained using the vibration-based CM have located the causes of high vibration levels. Due to the constructional solution of the investigated reduction gear, a replacement of the entire S_2 and S_3 shafts with the belonging gears and roller bearings has been necessary. After replacing these elements, it has been possible to visually see some of the problems identified in the frequency spectrums. The photos of the replaced elements are shown in Figs 6 and 7.

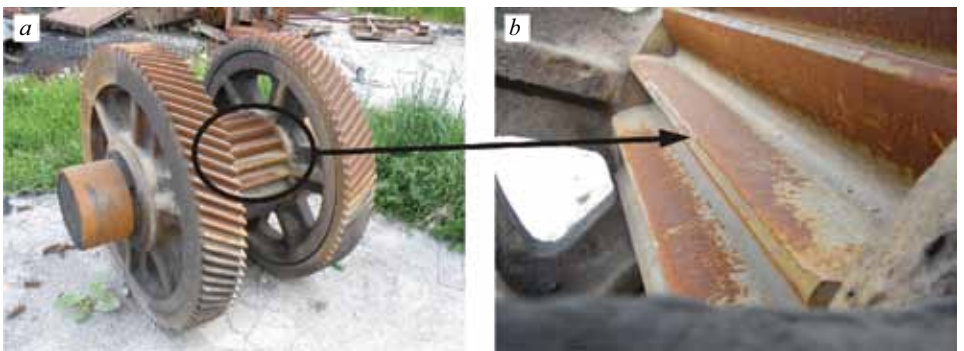


Fig. 6. Photographs of the shaft S_2 an overall view (a) and a pinion detail, surfaces where plastic flow occurs (b)

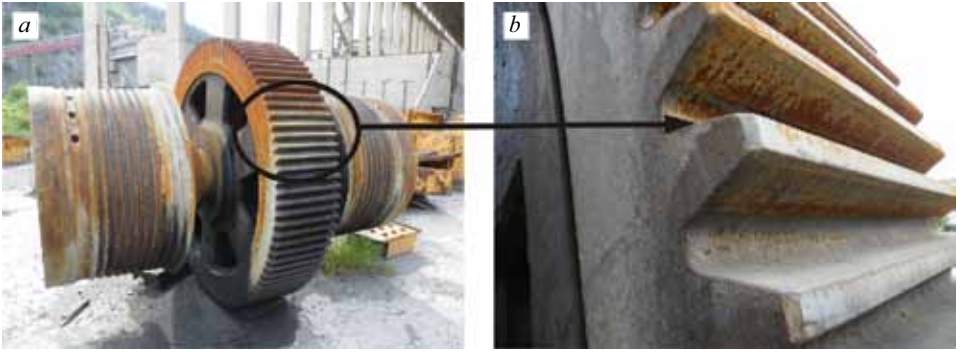


Fig. 7. Photographs of the shaft S_3 an overall view (a) and a gear wheel detail shows that there are no visible destructions (b)

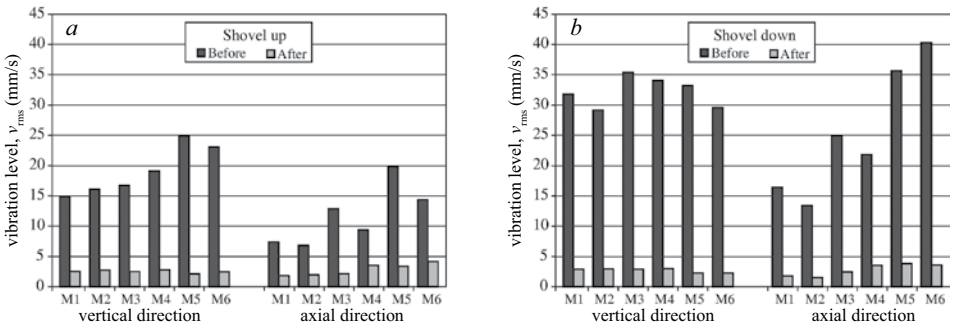


Fig. 8. Comparison of vibration levels before and after repair of the reduction gear, during the shovel up period (a) and during the shovel down period (b)

The plastic flow of the pinion can be noticed in Fig. 6. This failure mechanism is caused by high contact stresses and the rolling and sliding action of the mesh¹⁸. It is a surface deformation resulting from the yielding of the surface and subsurface material, and it is usually associated with softer gear materials, although it can occur in heavily loaded case-hardened gears as well. It has been noted by visual inspection that the degradation of the tooth gear wheel is much less pronounced (see Fig. 7). After replacing the above mentioned elements, vibration measurements on the reduction gear have been repeated through final stage investigations. Comparative illustrations of the measurement results before and after repair of the reduction gear are shown in Fig. 8.

Final stage investigations have shown that after carrying out corrective actions, the vibration level of the reduction gear has been significantly reduced (depending on the measurement point by 2.5 to 14.5 times).

COST ANALYSIS

In order to solve the problem of the high vibration levels of the reduction gear, there were two possible solutions. The first choice has been related to the replacement of the full reduction gear, while the second option has involved a replacement of the reduction gear elements which have been identified as critical, according to the vibration-based CM. A comparative cost analysis of the two choices is given in Table 4.

Table 4. Cost analysis of the two possible choices

1st choice		2nd choice	
description	cost (€)	description	cost (€)
Complete reduction gear replacement	220000	Replacement of S2 and S3 shaft	87000
Labour cost 1 engineer + 4 technicians 16 hours × 5 days × 25 €/h + 16 hours × 5 days × 12 €/h × 4	5840	Labour cost 1 engineer + 4 technicians 16 hours × 5 days × 25€/h + 16 hours × 5 days × 12 €/h × 4	5840
Crane 60 tons 3 days × 12 hours × 300 €/h	10800	Crane 40 tons 3 days × 12 hours × 250 €/h	9000
–	–	Vibration based CM	1500
Overall costs	236640	Overall costs	103340

The largest share of the total costs for both solutions has been the value of parts that need to be replaced during the repair. The labour costs are identical in both situations, because these two choices require the same number of people to be involved and the same commitment time. The first choice requires greater engagement of the crane capacity, bringing a slightly higher cost. The smallest shares of the total costs are the vibration-based CM. This assumption is taken in accordance with the first solution which does not need failure analysis. The cost analysis shows that the second choice brings savings of 56% of assets, respectively about 133300 € in the total amount.

CONCLUSIONS

The failure analysis of a massive excavator reduction gear has been realised through vibration-based CM. A process of vibration signal digital resampling using a cubic spline interpolation algorithm has been used to collect information in order to determine the condition of the investigated reduction gear. In the obtained frequency spectrum, discrete components such as gear mesh frequencies, related sidebands and harmonics are clearly separated. The research results have located the causes of high vibration levels and also assisted in maintenance decision making. Estimated savings that have been achieved on this occasion are 56%. After the replacement of the damaged elements, a vibration re-measurement was carried out on the reduction gear. On that occasion, we found that overall vibration levels were reduced signifi-

cantly, by 2.5 to 14.5 times. The results obtained with the vibration-based CM have been confirmed by the visual inspection of the replaced elements.

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REFERENCES

1. R. K. MOBLEY: An Introduction to Predictive Maintenance. 2nd ed., Butterworth – Heinemann, Elsevier, Woburn MA, 2002.
2. S. KARABAY, I. UZMAN: Importance of Early Detection of Maintenance Problems in Rotating Machines in Management of Plants: Case Studies from Wire and Tyre Plants. *Eng. Fail. Anal.*, **16**, 212 (2009).
3. K. S. A. JARDIN, D. LIN, D. BANJEVIC: A Review on Machinery Diagnostics and Prognostics Implementing Condition-based Maintenance. *Mech. Syst. Signal Process.*, **20**, 1483 (2006).
4. P. GIRDHAR, C. SCHEFFER: Practical Machinery Vibration Analysis and Predictive Maintenance. Newnes, Elsevier, 2004.
5. J. P. AMEZQUITA-SANCHEZ, E. CABAL-YEPEZ, R. J. ROMERO-TRONCOSO, R. A. OSORNIO-RIOS, A. GARCIA-PEREZ: Determination of System Frequencies in Mechanical Systems during Shutdown Transient. *J. Sci. Ind. Res.*, **69**, 415 (2010).
6. K. R. FYFE, E. D. S. MUNCK: Analysis of Computed Order Tracking. *Mech. Syst. Signal Process.*, **11**, 187 (1997).
7. P. TODOROVIC, B. JEREMIC, I. MACUZIC, A. BRKOVIC, U. PROSO: Vibration Analysis of Cracked Rotor during Run-up. *Tribology in Industry*, **30**, 55 (2008).
8. A. BESLAK, J. FLASKER: Detecting Crack in the Tooth Root of Gears. *Eng. Fail. Anal.*, **14**, 1466 (2007).
9. S. GADE, H. HERLUFSEN, H. KONSTANTIN-HANSEN, N. J. WISMER: Order Tracking Analysis. Technical Review No 2, Bruel & Kjaer, Naerum, 1995.
10. J. R. BLOUGH: A Survey of DSP Methods for Rotating Machinery Analysis, what is Needed, what is Available. *J. Sound Vib.*, **262**, 707 (2003).
11. K. S. WANG, P. S. HEYNS: Application of Computed Order Tracking, Vold–Kalman Filtering and EMD in Rotating Machine Vibration. *Mech. Syst. Signal Process.*, **25**, 416 (2011).
12. R. B. RANDALL: Detection and Diagnosis of Incipient Bearing Failure in Helicopter Gearboxes. *Eng. Fail. Anal.*, **11**, 177 (2004).
13. B. TADIC, P. M. TODOROVIC, D. J. VUKELIC, B. M. JEREMIC: Failure Analysis and Effects of Redesign of a Polypropylene Yarn Twisting Machine. *Eng. Fail. Anal.*, **18**, 1308 (2011).
14. P. D. McFADDEN: Interpolation Techniques for Time Domain Averaging of Gear Vibration. *Mech. Syst. Signal Process.*, **3**, 87 (1989).
15. G. DAHLQUIST, A. BJORCK: Numerical Methods in Scientific Computing. Vol. I. Society for Industrial and Applied Mathematics, Philadelphia, 2008.
16. G. H. GOLUB, C. F. Van LOAN: Matrix Computations. 3rd Ed. The Johns Hopkins University Press, Baltimore and London, 1996.
17. R. K. MOBLEY: Root Cause Failure Analysis. Butterworth – Heinemann, Elsevier, Woburn MA, 1999.
18. M. POPOVIC, Z. JUGOVIC, R. SLAVKOVIC: The Concept of Function Virtual Prototype in the Design of Excavator Cutting Teeth. *Tribology in Industry*, **31**, 37 (2009).

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