

SOLUTION OF FLUID FLOW THROUGH THE LEFT HEART VENTRICLE

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Abstract:

Finite element method (FEM) has proved to be very successful in solving complex problems in engineering. In medicine, FEM found a special application in simulation of real problems and their solution while eliminating the need of learning from examples. In this paper, FEM was used with viscous fluid dynamics to simulate the function of left heart ventricle. Theoretical fundamentals of FEM and heart function were given. The paper represents illustrative example for educational purposes.

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1. INTRODUCTION

Finite element method (FEM) represents one of the most used methods in solving and simulating different problems not only in different fields of engineering but also in medicine. Earliest examples of the FEM usage date from the early forties of the 20th century when Alexander Hrennikoff used this method to describe material elasticity [1] while Richard Courant used it to solve the problem of equilibrium and vibrations [2]. Even though the most of the scientific society recognised the potential of this method right away, FEM achieved its first big success in sixties and seventies of the 20th century with the work of John Argyris, Ray William Clough and Olgierd Zienkiewicz afterwards with the work of Ernest Hinton and Bruce Iron [3].

While these were pioneers of FEM analysis, in Serbia, we should point out the work of Milos Kojic who brought this method in Serbia in the early eighties of the 20th century. In addition to numerous publications in this field, professor Kojic was the founder of the software package PAK [4] which

represents a FEM solver for the analysis of constructions and which was used in this paper.

Besides in engineering, FEM has found a big application in the analysis of the complex problems in medicine and their simulation through the direct use of viscous fluid dynamics. Even though this method can be used to simulate real diseases, it can also be used for their validation and representation of the methods used to solve problems associated with these diseases. From the simulation of a stent deployment in blood vessels [5] to the simulations of heat diseases [6], the finite element method shows a rising potential even after 80 years since it was first used.

This paper shows an example of the FEM usage in the software package PAK in the field of bioengineering and viscous fluid dynamics regarding the function of the left heart ventricle. The paper is divided in five sections that present theoretical fundamentals for the function of the human heart in medicine and that are important for understanding of the problem studied in this paper. They also represent theoretical fundamentals for

the FEM in viscous fluid dynamics used in PAK. The section three shows methods and steps used to make and calibrate the model of the left heart ventricle, while the section four gives the test results. Finally, the section five gives the conclusion and describes a future line of work in this field together with the summary of the paper.

2. HEART MUSCLE

In order to understand the problems that occur in engineering while we are trying to simulate anatomy processes, we first have to understand the actual anatomy. The heart muscle consists of a left and right heart ventricle and the correspondent atriums, as shown in Figure 1.

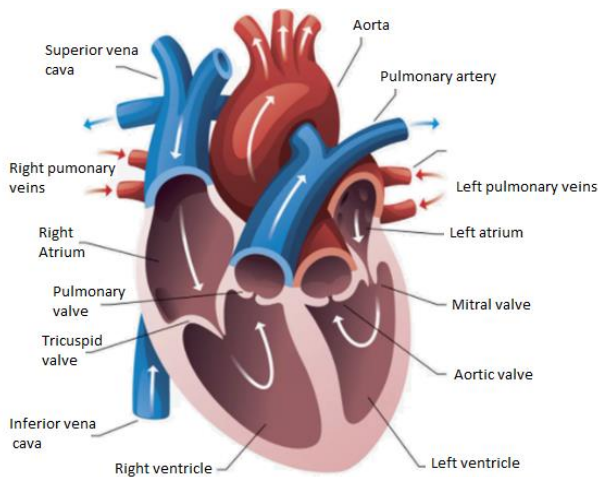


Fig.1. Heart muscle [7]

While the right ventricle has the role to transport blood to the lungs, the left ventricle has a more important role to transport blood to the entire system of organs.

If we look at the right ventricle, we can see that blood enters the heart through the superior and inferior vena cava, as it can be seen in Figure 1. When blood enters the right atrium, it contracts and blood enters the right ventricle through the tricuspid valve. When the right ventricle reaches its maximum capacity, the tricuspid valve closes in order to prevent more blood from entering the right ventricle but also to prevent it from exiting the ventricle while it is contracting. With the contraction of right ventricle, blood flows to the pulmonary artery through the pulmonary valve at the end of which blood oxidation takes place.

After the oxidation process in the lungs, blood rich in oxygen flows into the left atrium through the pulmonary veins. Like the right ventricle, the left atrium contracts and blood enters the left ventricle

(LV) through the mitral valve and, when the left ventricle is full, the mitral valve closes. It closes for the same reason as the tricuspid valve. With the contraction of the left ventricle, blood exits the left ventricle through the aortic valve and enters the aorta which transports blood to other organs.

The proper function of the left heart ventricle is important because when the left ventricle does not work properly we do not get the right amount of blood in our organs which causes problems. A lot of scientists focus on cardiovascular problems because there is a large number of different cardiovascular illnesses.

3. FEM AND FLUID DYNAMICS

The continuity equation represents an analytical representation of the mass conservation law which can be applied to the elemental mass of a fluid part dm , and on the final mass m . According to this law, the mass of a fluid part does not change during its motion in a continuous current field. By applying Reynolds theorem on the mass conservation law [8]:

$$\frac{Dm}{Dt} = \frac{D}{Dt} \int_V \rho dV = \int_V \left(\frac{D\rho}{Dt} + \rho \frac{\partial v_i}{\partial x_i} \right) dV = 0 \quad (1)$$

for the arbitrary control volume V we get the continuity equation at a point as:

$$\frac{D\rho}{Dt} + \rho \frac{\partial v_i}{\partial x_i} = 0 \quad (2)$$

In fluid dynamics, Navier-Stokes equations are used to describe the motion of viscous fluids. They represent a system of partial differential equations derived from Newton's second law.

Observe the fluid volume at a given point in time t (Figure 2). Here, the external forces are represented as surface forces f^s per unit area and as volume forces f^B per unit of mass.

Based on the equation of momentum change, [8], we get:

$$\frac{D}{Dt} \int_V \rho v dV = \int_V b dV + \int_S p dS \quad (3)$$

By applying Reynolds theorem [8], the equation 3 becomes:

$$\int_V \rho \frac{Dv}{Dt} dV = \int_V f^B dV + \int_S f^S dS \quad (4)$$

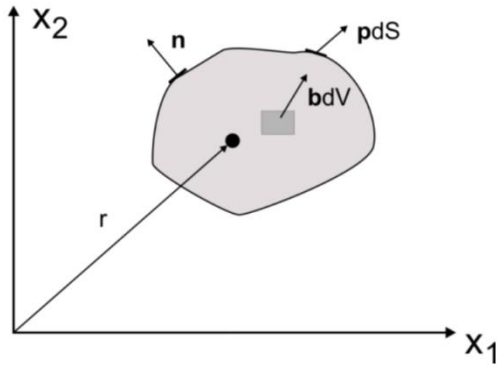


Fig.2. Elemental volume of a fluid with volume and surface forces [9]

Now it is possible to apply Gauss and Cauchy theorem [10] for connection between the surface and the volume integrals:

$$\int_V \rho \frac{Dv_i}{Dt} dV = \int_V f_i^B dV + \int_V \frac{\partial \sigma_{ij}}{\partial x_j} dV \quad (5)$$

$$\sigma_{ij} = -p\delta_{ij} + 2\mu\dot{e}_{ij} \quad (6)$$

$$\dot{e}_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \quad (7)$$

Then, using the constitutive relation for Newton’s fluid, the equation 6, and the tensor of deformation velocity, equation 7, where p stands for the fluid pressure, μ stands for the coefficient of dynamic viscosity and \dot{e} represents the tensor of deformation velocity, we get:

$$\int_V \rho \frac{Dv_i}{Dt} dV = \int_V f_i^B dV + \int_V \left(-\frac{\partial p}{\partial x_i} + \mu \left(\frac{\partial^2 v_i}{\partial x_j \partial x_j} + \frac{\partial^2 v_j}{\partial x_j \partial x_i} \right) \right) dV \quad (8)$$

While the control volume V is arbitrary, the equation 8 can be written in a differential form through which we get a standard Navier-Stoks equations for incompressible viscous flow:

$$\rho \left(\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} \right) = -\frac{\partial p}{\partial x_i} + \mu \left(\frac{\partial^2 v_i}{\partial x_j \partial x_j} + \frac{\partial^2 v_j}{\partial x_j \partial x_i} \right) + f_i^B \quad (9)$$

However, for the problem of the blood flow through the left heart ventricle, described in this paper, the density change does not occur and the volume forces are neglected. Thus, the continuity equation (2) and differential Navier-Stocks equations (9) are represented as:

$$-\mu \nabla^2 v_l + \rho (v_l \cdot \nabla) v_l + \nabla p_l = 0 \quad (10)$$

$$\nabla v_l = 0 \quad (11)$$

Where v_l is the blood flow velocity, p_l is the pressure, μ is the coefficient of dynamic viscosity of blood and ρ is the density of blood.

Besides the continuity equation and differential Navier-Stocks equations (10 and 11 respectively), the transfer of mass is modelled using the equation for convective diffusion:

$$\nabla \cdot (-D_l \nabla c_l + c_l v_l) = 0 \quad (12)$$

Where D_l is the coefficient of diffusion and c_l is blood concentration.

After derivation, the finite element equation takes the following form:

$$\begin{bmatrix} \mathbf{M}_v & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{v}} \\ \dot{\mathbf{p}} \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{vv} + \mathbf{K}_{\mu v} & \mathbf{K}_{vp} \\ \mathbf{K}_{vp}^T & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{v} \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} \mathbf{R}_B + \mathbf{R}_S \\ \mathbf{0} \end{Bmatrix} \quad (13)$$

4. METHODOLOGY

The problem of the blood flow through the left heart ventricle was analysed using BioIRC’s in-house user interface software CAD, which was adjusted to this specific problem.

In order to solve this problem, there are several steps to take. We need to:

- Create geometry of the left ventricle
- Create the boundary conditions and the input parameters for the calculation
- Run the calculation and
- Read the results.

To create geometry, we first need to check the box Enable above the button **LV Param** as shown in Figure 3.

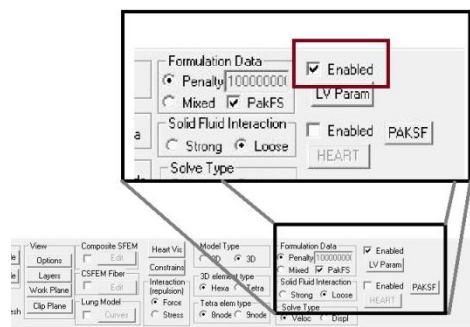


Fig.3. Enabling the geometry creation

After enabling the geometry creation, and after clicking the button **LV Param**, a window with parameters is shown. It is necessary to set the

geometry parameters of the left ventricle in this window. Figure 4 shows the parameters set for the calculation used in this paper.

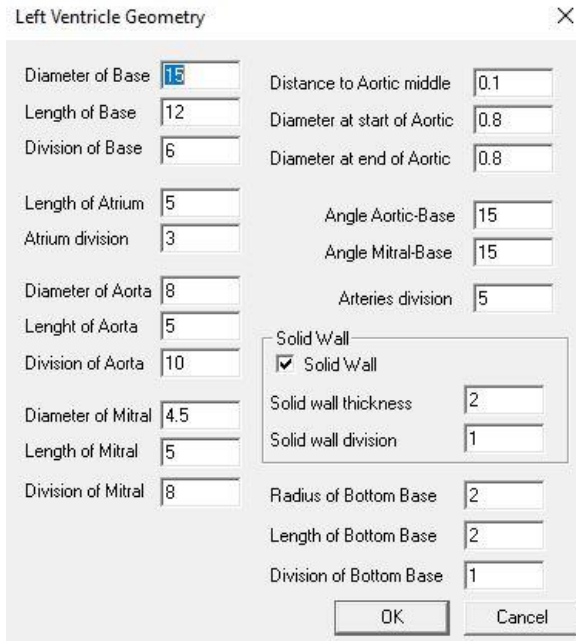


Fig.4. Geometry parameters for the calculation

After confirming the parameters by clicking on **OK** it is necessary to click on **Generate** button, located in top right corner, in order to generate the geometry with the selected parameters. Figure 5 shows the geometry that was created this way.

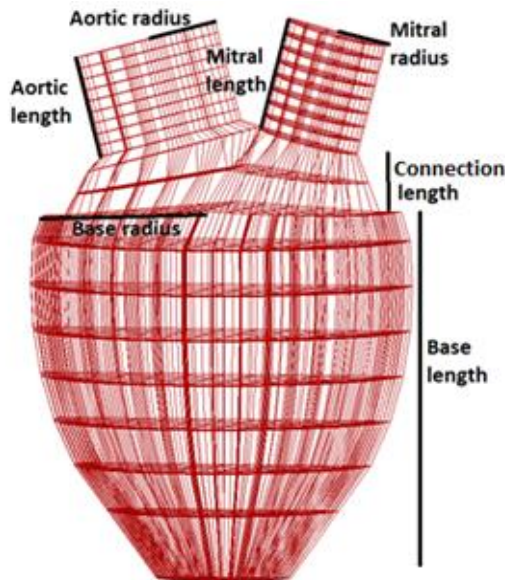


Fig.5. Generated geometry of the left ventricle

Now, we need to set the input parameters for the calculation. In order to do this, we first need to specify the type of the model, to enable creation of

the input parameters and finally to set them. All of these steps are shown in Figure 6.

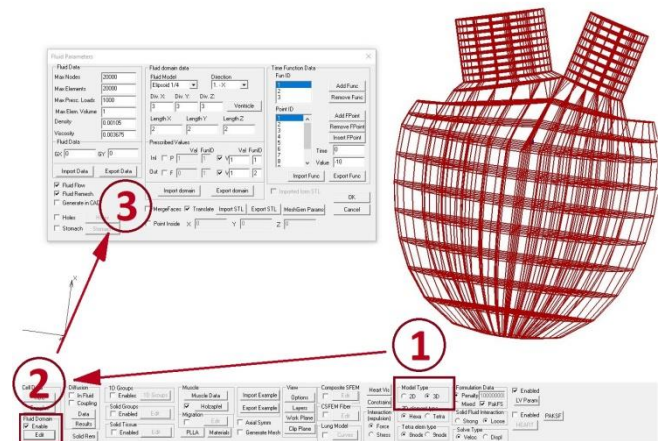


Fig.6. Setting the input parameters

After setting the input parameters, i.e. the Fluid Data and Fluid Domain data, we need to describe the input functions for the blood flow through the mitral valve (function 1) and the blood flow through the left ventricle and the aortic valve (function 2). The values of these functions are shown in Table 1.

Table 1. Functions used for the calculation

Step	Parameter	Func. 1	Func. 2
1	Time	0	0
	Value	-10	0
2	Time	0.1	0.1
	Value	-20	0
3	Time	0.2	0.2
	Value	-30	0
4	Time	0.3	0.3
	Value	-40	0
5	Time	0.4	0.4
	Value	-44	0
6	Time	0.5	0.5
	Value	-30	0
7	Time	0.6	0.6
	Value	0	0.5
8	Time	0.7	0.7
	Value	0	1
9	Time	0.8	0.8
	Value	0	12
10	Time	0.9	0.9
	Value	0	8
11	Time	1	1
	Value	0	2

As it can be seen in Table 1, the input functions imitate the left ventricle function as it was described in the section 2. Blood is pumped through the mitral valve inside the left ventricle while being careful not to overfill the left ventricle. This can be

seen from the function 1 step 6 where the blood's velocity decreases. The mitral valve closes and the blood is pumped out through the aortic valve as shown in Figure 5.

In the end, it is necessary to set the number of steps for the calculation by clicking on **Time Steps** button in the top right corner. Ten steps with a time of 0.1 was chosen to ensure that all phases of the load itself have passed.

5. RESULTS

In this section, we will show and explain the results regarding the shear stress, displacement and blood flow velocity. Figure 7 shows the results for shear stress and the location of its maximum.

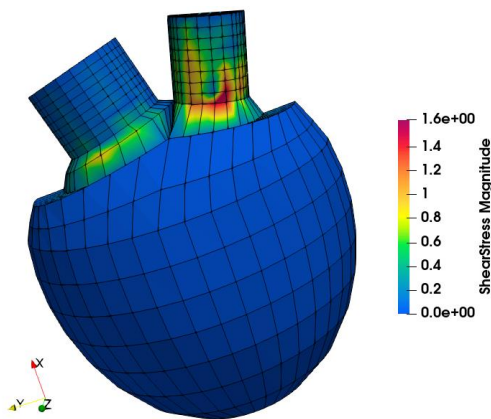


Fig.7. Wall shear stress

As it can be seen in Figure 7, the location of the maximum wall shear stress is on the wall of the mitral valve. This is due to the fact that the biggest change of diameters is at that specific place which represents the entrance to the left ventricle. The result shows that the mitral valve experiences the highest amount of strain during the blood flow through the left ventricle.

Image 8 shows the amount of displacement that the heart does during each contraction.

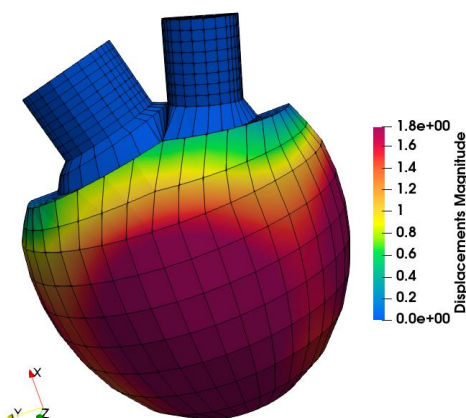


Fig.8. Effective displacement of the left ventricle

The displacement shown in Figure 8 can be explained through anatomy. When the heart contracts, the bottom half of the ventricles move up in order to pump the blood out. This effect is shown in Figure 8.

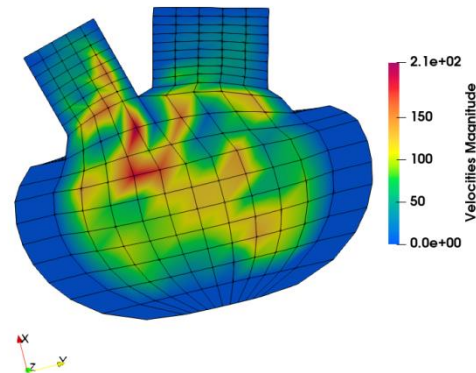


Fig.9. Values for the blood flow velocity inside the left ventricle during contraction

Figure 9 shows the velocity that blood experiences during each contraction. The blood flows through the mitral valve at high speeds inside the left ventricle. When the left ventricle reaches its critical volume it pumps the blood out through the aortic valve at speeds slower than the ones through the mitral valve. This is easily explained with a systolic and diastolic blood pressure. Since the systolic pressure is always higher than diastolic, we can conclude that the blood velocity during the intake will be higher than during the outtake.

6. CONCLUSION

This paper has shown the approach to the FEM analysis in the field of medicine and viscous fluid dynamics. A detailed description of theoretical foundations in the field of medicine and the field of finite element method in viscous fluid dynamics were shown in sections 2 and 3. The methods used for the calculation of the blood flow through the left ventricle was given in the section 4.

The results given in section 5 show the similarity between our calculations and the anatomy of the problem. The results were carefully explained for the parameters of the wall shear stress, effective displacement and the values of the blood flow velocity inside the left ventricle. The calculation was done using PAK solver while the model for the calculation was made using the in-house CAD software.

Our future work will be focused on the calculation of the blood flow through the human heart.

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